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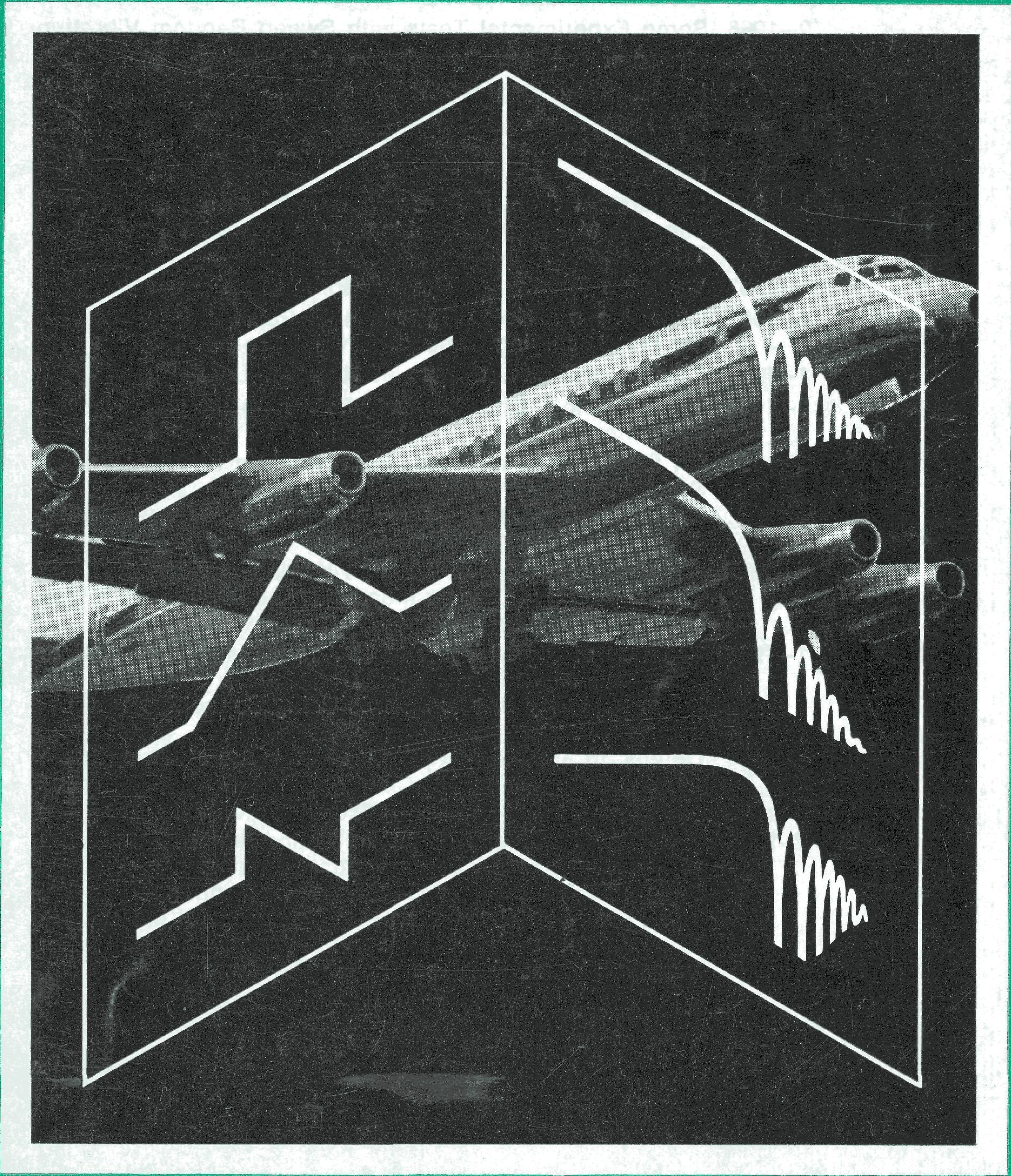

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Frequency Analysis of Single Pulses

by

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ABSTRACT

It is often difficult to do frequency analysis of single pulses at the time they occur.
A possible solution is to record them on an FM modulated tape-recorder and to do the frequency analysis in the laboratory. It is essential to be conscious of the limitation of tape-recorders especially of the tape-recorder used. The possibilities of misinterpretations are high. However correctly applied a suitable tape-recorder will give the possibility of a really extensive frequency analysis.
In the paper, there will be shown the results of frequency analysis using closed tape loops on a B & K FM modulated tape-recorder type 7001.
The influence of frequency transformation of the signal and the influence of the choice of different tape-loop lengths on the possibility of achieving optimum results is discussed. The choice of different frequency analyzers is discussed.
In the paper, there will be mentioned some practical precautions to take when using closed tape loops. An electronic circuit, whose purpose is to eliminate unwanted signals such as splice pulses on the tape loops is described.

SOMMAIRE

Il est souvent difficile d'effectuer une analyse de fréquence d'impulsions solitaires, au moment où elles se produisent.
Une solution possible consiste à les enregistrer sur un enregistreur à modulation FM et à faire ensuite, en laboratoire, l'analyse de l'enregistrement. Il est très important d'avoir conscience des limitations apportées par les enregistreurs, spécialement par celui qu'on utilise. Les risques de mauvaise interprétation sont nombreux. Cependant, correctement utilisé, un enregistreur à ruban adéquat procurera la possibilité d'une analyse de fréquence réellement poussée.
On montre dans l'article les résultats de l'analyse de fréquence obtenue en utilisant des boucles fermées de ruban sur un enregistreur B & K à modulation FM, type 7001.
L'influence d'une transformation de fréquence du signal et l'influence du choix de différentes longueurs de boucle de ruban sur la possibilité d'obtenir les meilleurs résultats sont examinées. La question du choix de différents analyseurs de fréquence est également discutée.
L'article mentionne quelques précautions pratiques à prendre lorsqu'on utilise des boucles fermées de ruban. Un circuit électronique destiné à éliminer les signaux non désirés, telles les impulsions qui se produisent à l'endroit où les boucles sont collées.

ZUSAMMENFASSUNG

Für die Frequenzanalyse von Einzelimpulsen bietet die FM-Aufzeichnung dieser Impulse auf Magnetband und die Bandschleifentechnik eine Möglichkeit.
Der Verfasser zeigt die Ergebnisse einer Analyse auf, für die er das B & K Bandgerät 7001 benutzte. Er diskutiert den Einfluß der Frequenztransformation und den unterschiedlicher Bandschleifenlängen. Auch geht er auf Frequenzanalysatoren ein. Einige praktische Hinweise für die Anwendung der Bandschleifentechnik werden gegeben und eine Schaltung wird beschrieben um Störsignale, wie das Klebstellengeräusch bei der Bandschleife, zu unterdrücken.

Introduction

In recent years interest in measurement and analysis of single pulses has increased considerably. This is due to several reasons of which a few could be mentioned:

Air traffic will soon employ super-sonic air-crafts which will subject an increasing number of populated areas to annoying and destructive pressure waves, "sonic booms".

Increasing refinement in mechanical engineering demands a better knowledge of the environment and of the response of materials and systems to this environment.

In some industries the response of materials to controlled single pulses is used to indicate when the processing of the materials can be discontinued.

In this paper different ways of frequency analyzing FM tape recordings of "sonic booms" are discussed but the methods described can easily be applied to the analysis of other single pulses.

Fourier Integral Transformation and Fourier Series

A single pulse $f(t)$ can be described in the frequency domain by the Fourier integral:

$$F(f) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\, dt \tag{1}$$

The resultant continuous functions for the following single pulses are computed in Appendix A:

1. A rectangular single pulse (see Fig. 1).

$$F(f) = AT\frac{\sin \pi Tf}{\pi Tf} \tag{1.1}$$

2. A triangular single pulse (see Fig. 2).

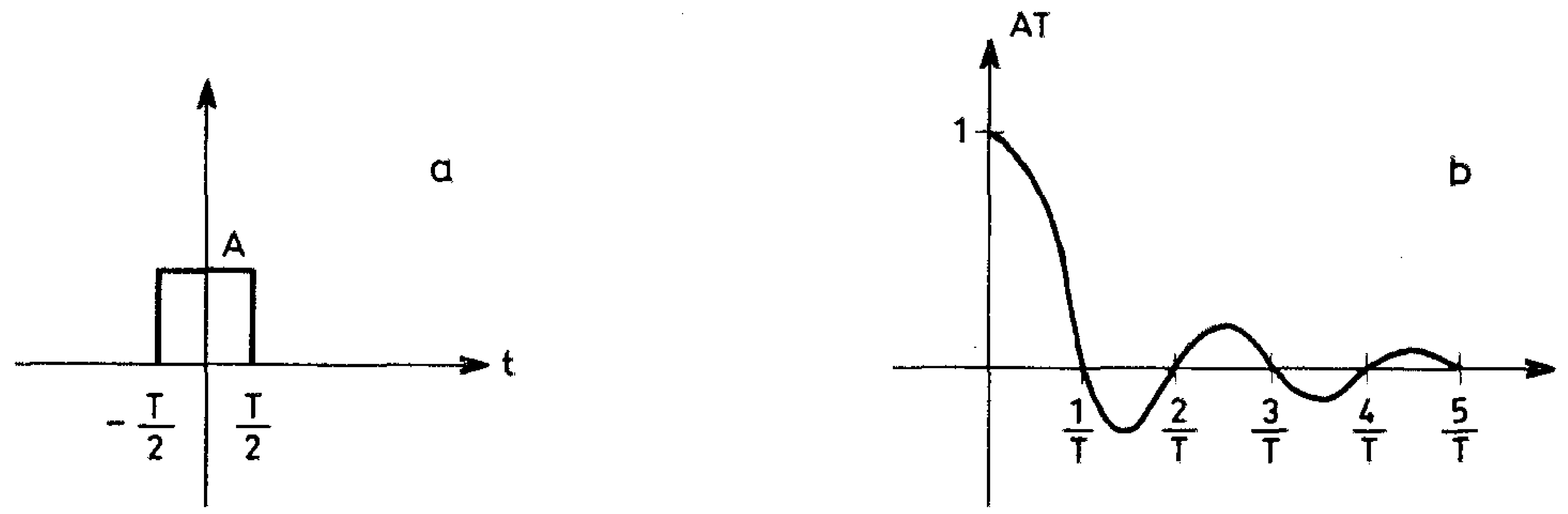
$$F(f) = AT\left(\frac{\sin \pi Tf}{\pi Tf}\right)^2 \tag{1.2}$$

3. An N-shaped single pulse, sonic boom (see Fig. 3).

$$F(f) = j\,AT\,\frac{1}{\pi Tf}\left(\frac{\sin \pi Tf}{\pi Tf} - \cos \pi Tf\right) \quad \text{for } \tau = 0 \tag{1.3}$$

$$F(f) = j\,AT\,\frac{1}{\pi Tf}\,\frac{T}{2\,\tau}\left(\frac{\sin \pi T\,(1-\frac{2\,\tau}{T})\,f}{\pi T\,(1-\frac{2\,\tau}{T})\,f} - \frac{\sin \pi Tf}{\pi Tf}\right) \quad \text{for } \tau > 0 \tag{1.4}$$

Measurement of such frequency functions might be done by using a parallel analyzer containing a number of parallel filters covering the interesting frequency band. A similar result would be obtained by playing back a tape-recording of the pulse in question once into each filter of a normal sequential analyzer.



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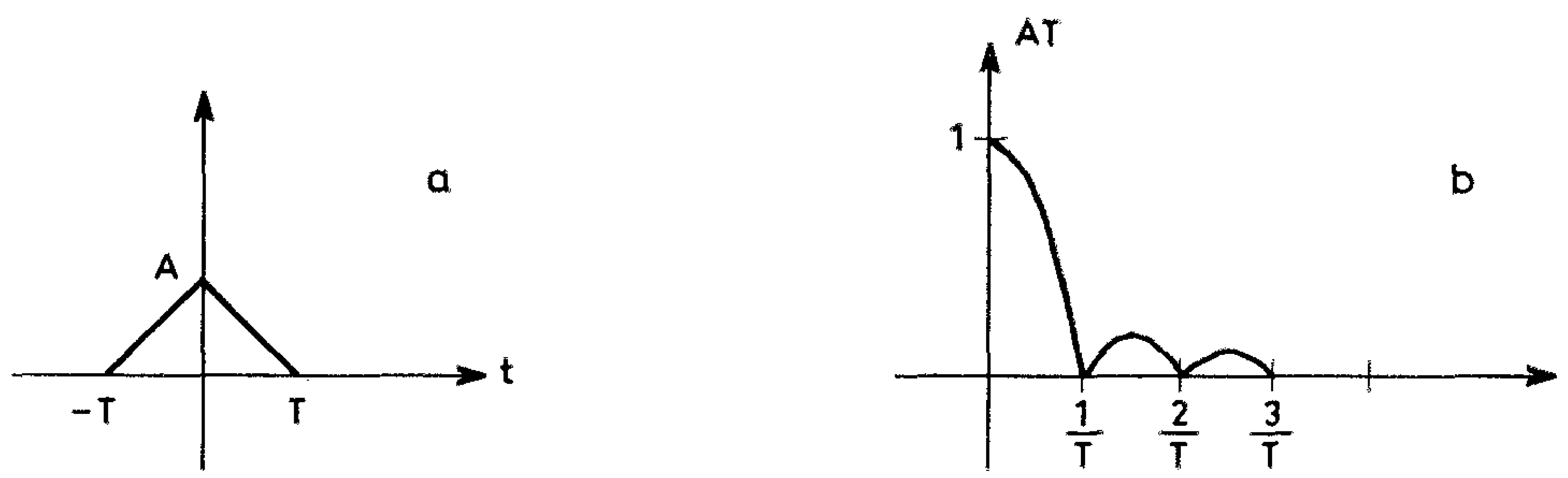
$$f(t) = \begin{cases} A \text{ for } -\frac{T}{2} < t < \frac{T}{2} \\ O \end{cases} \qquad F(f) = AT\,\frac{\sin \pi Tf}{\pi Tf}$$

Fig. 1. Single rectangular pulse and its frequency spectrum.



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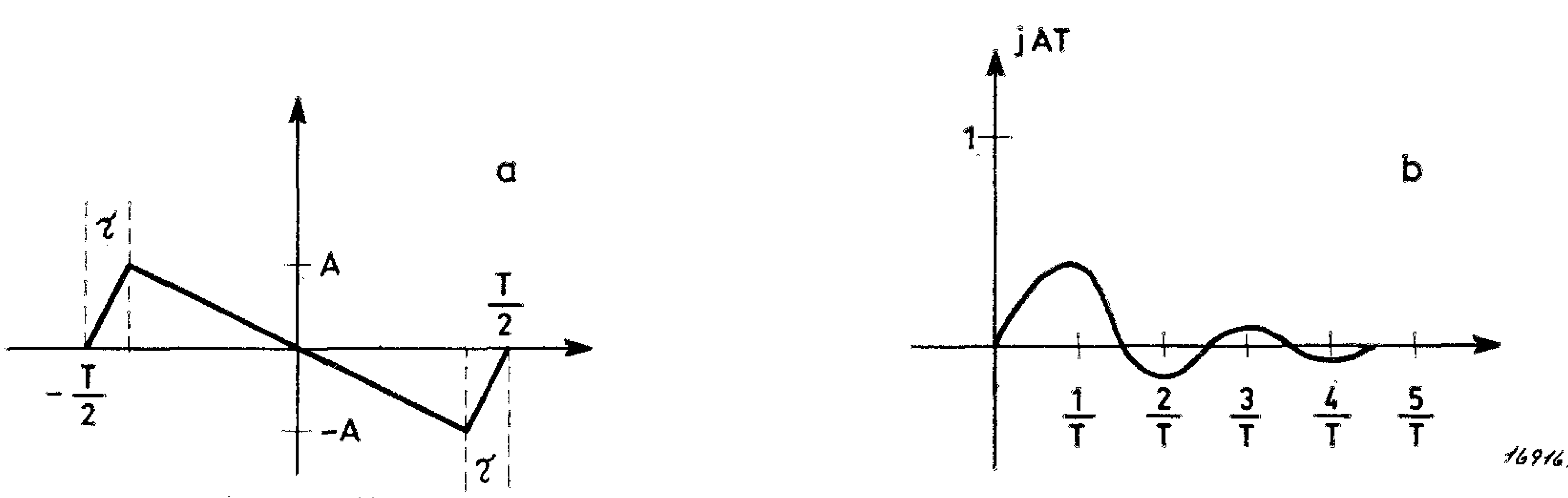
$$f(t) = \begin{cases} A\,(1+\frac{t}{T}) \text{ for } -T < t < 0 \\ A\,(1-\frac{t}{T}) \text{ for } 0 < t < T \\ O \end{cases} \qquad F(f) = AT\left(\frac{\sin \pi Tf}{\pi Tf}\right)^2$$

Fig. 2. Single triangular pulse and its frequency spectrum.



$$f(t) = \begin{cases} A\,\dfrac{t+\frac{T}{2}}{\tau} \text{ for } \frac{T}{2} < t < -\frac{T}{2}+\tau \\ -A\,\dfrac{t}{\frac{T}{2}-\tau} \text{ for } -\frac{T}{2}+\tau < t < \frac{T}{2}-\tau \\ A\,\dfrac{t-\frac{T}{2}}{\tau} \text{ for } \frac{T}{2}-\tau < t < \frac{T}{2} \\ O \end{cases}$$

$$F(f) = j\,AT\,\frac{1}{\pi Tf}\left(\frac{\sin \pi Tf}{\pi Tf} - \cos \pi Tf\right) \text{ for } \tau = 0$$

$$F(f) = j\,AT\,\frac{1}{\pi Tf}\,\frac{T}{2\tau}\left(\frac{\sin \pi T\left(1-\frac{2\tau}{T}\right) f}{\pi T\left(1-\frac{2\tau}{T}\right) f} - \frac{\sin \pi Tf}{\pi Tf}\right) \text{ for } \tau > 0$$

Fig. 3. Single N-shaped pulse and its frequency spectrum.

A useful and convenient method, however, is to record the pulse on an FM tape recorder, transfer it to a closed tape loop and to continuously play-back the recorded signal and thereby obtain a periodic signal with the cycle time T_B equal to the time the tape loop takes for a complete cycle in the tape recorder. The frequency functions of such periodic pulses are not continuous spectra but rather line spectra consisting of frequencies $n \times f_o = \frac{n}{T_B}$:

$$C_n = F(n \times f_o) = \frac{2}{T_B} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-j2\pi n f_o t}\, dt \qquad (2)$$

Comparison of (1) and (2) shows, however, that

$F(n \times f_o) = F(f) \times \frac{2}{T_B}$ at all frequencies where $F(n \times f_o)$ exists, that is at $f = n \times f_o$.

So by measuring $F(n \times f_o)$ then connecting the measuring points and multiplying by $\frac{T_B}{2}$ a picture of $F(f)$ is obtained. The measuring accuracy depends on having a sufficient number of measuring points.

The frequency functions measured will then be:

1. Rectangular pulses at intervals T_B

$$F(n \times f_o) = 2\,\frac{AT}{T_B}\,\frac{\sin \pi T n f_o}{\pi T n f_o} \qquad (2.1)$$

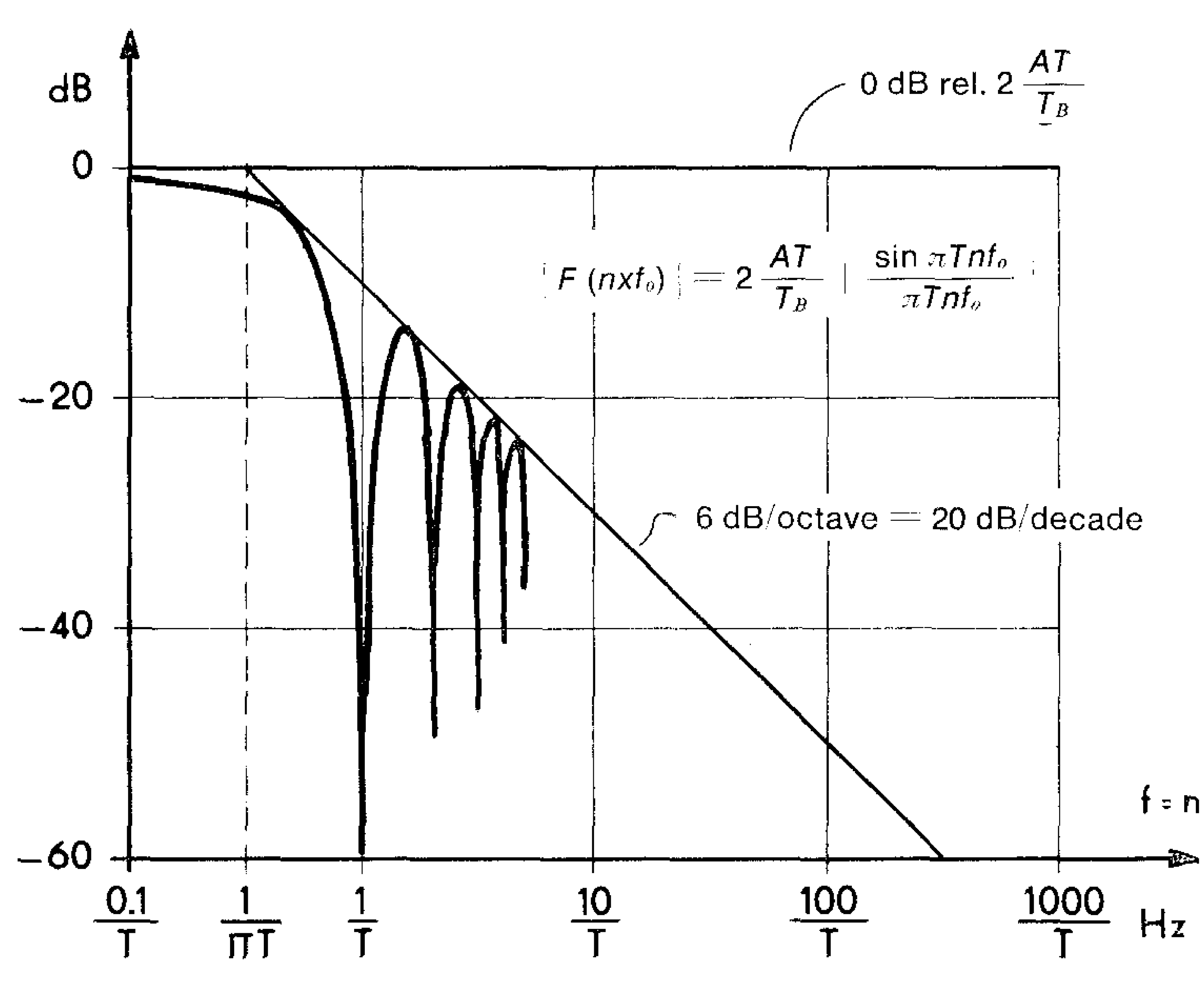


Fig. 4. Logarithmic presentation of frequency spectrum for rectangular pulses.

The function appears as points on the curve of Fig. 1 when the unit of the ordinate is $2\,\dfrac{AT}{T_B}$.

In Fig. 4 the numerical value of the function is shown plotted to a logarithmic scale.

2. Triangular pulses at intervals T_B

$$F(n \times f_o) = 2\,\frac{AT}{T_B}\left(\frac{\sin \pi T n f_o}{\pi T n f_o}\right)^2 \tag{2.2}$$

The function appears as points on the curve of Fig. 2 when the unit of the ordinate is $2\,\dfrac{AT}{T_B}$.

In Fig. 5 the numerical value of the function is shown plotted to a logarithmic scale.

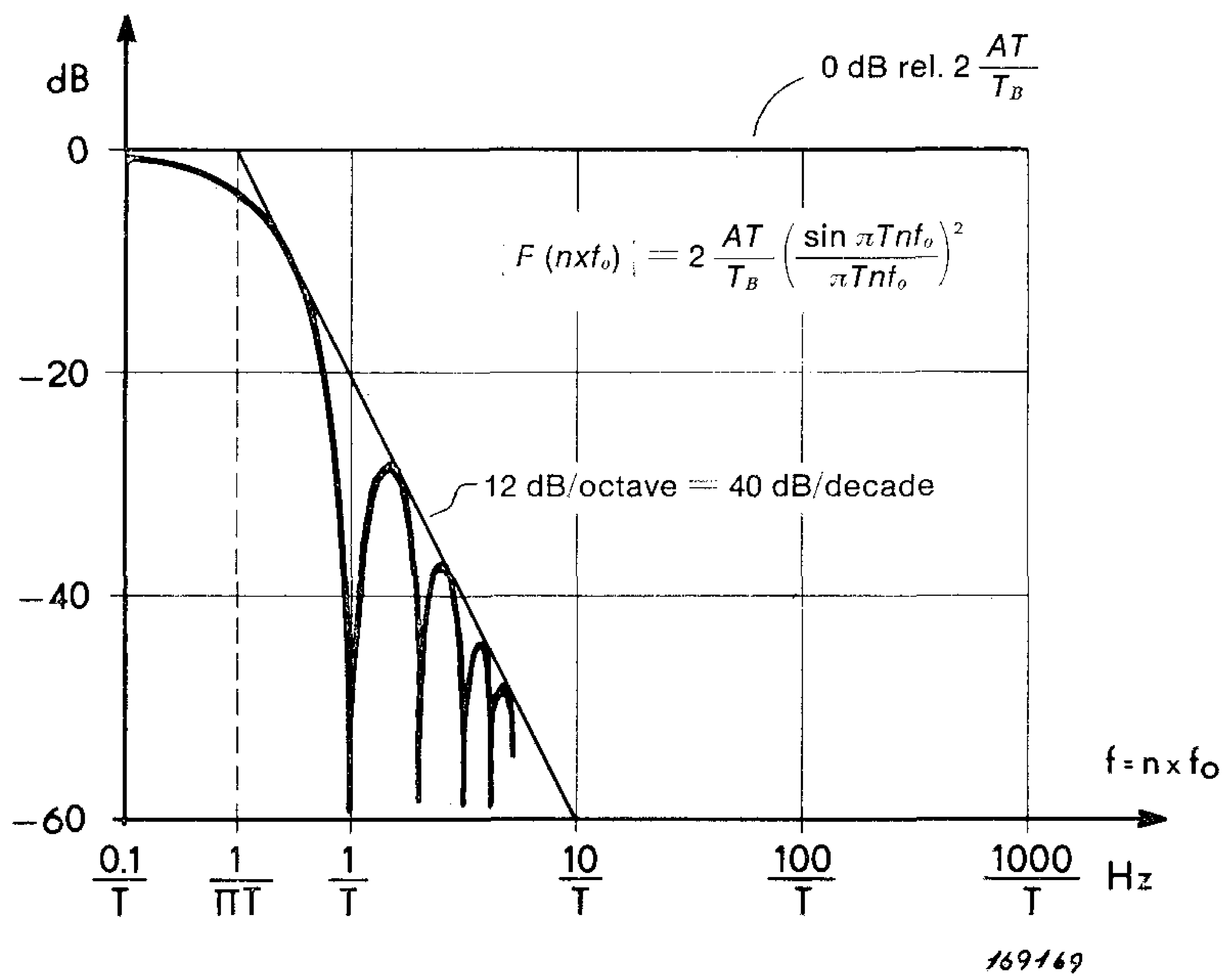


Fig. 5. Logarithmic presentation of frequency spectrum for triangular pulses.

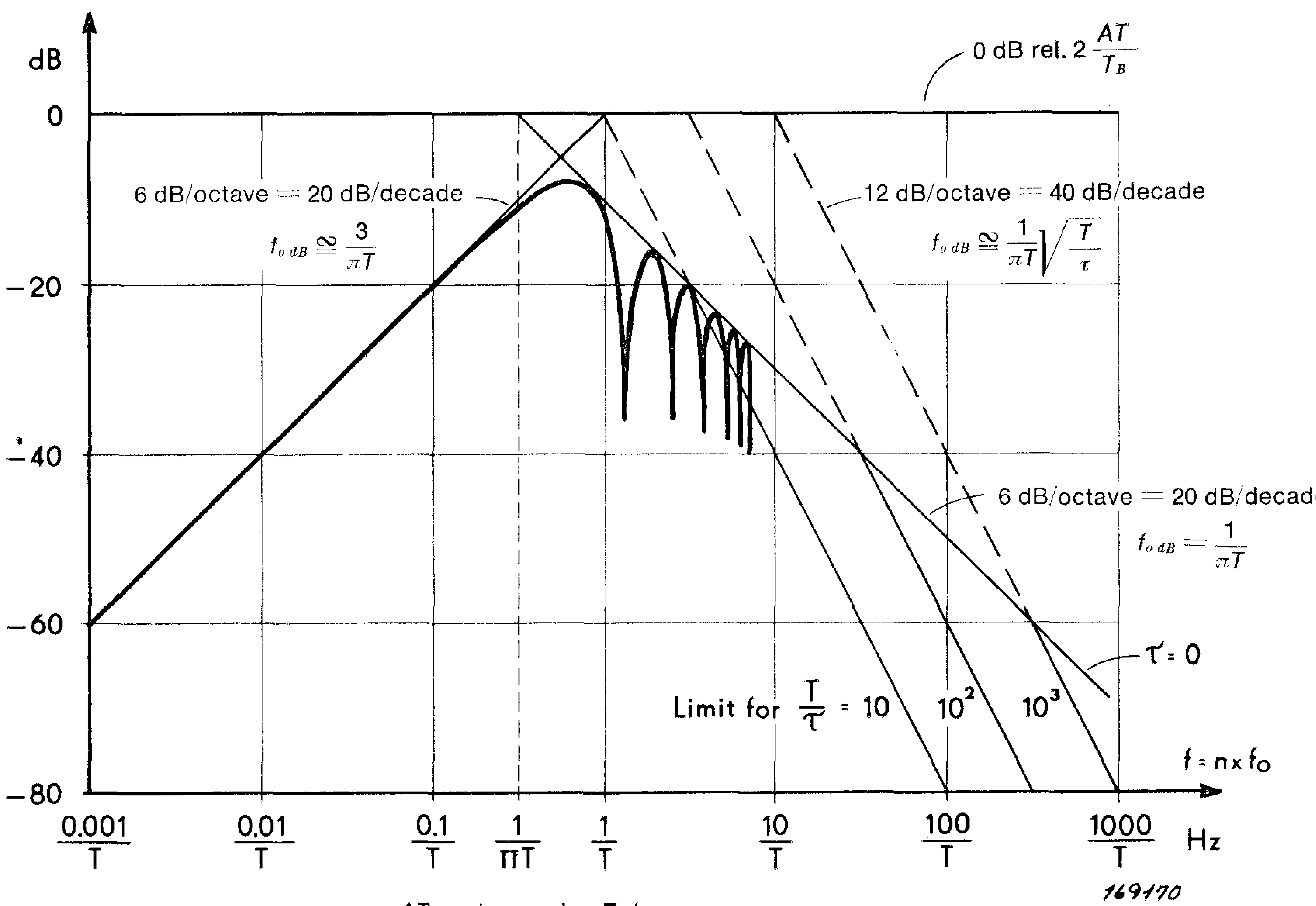
3. N-shaped pulses at intervals T_B

$$F(n \times f_o) = j\,2\,\frac{AT}{T_B}\,\frac{1}{\pi T n f_o}\left(\frac{\sin \pi T n f_o}{\pi T n f_o} - \cos \pi T n f_o\right) \tag{2.3}$$

$$F(n \times f_o) = j\,2\,\frac{AT}{T_B}\,\frac{1}{\pi T n f_o}\,\frac{T}{2\tau}\left(\frac{\sin \pi T\,(1-\frac{2\tau}{T})\,n f_o}{\pi T\,(1-\frac{2\tau}{T})\,n f_o} - \frac{\sin \pi T n f_o}{\pi T n f_o}\right) \tag{2.4}$$

The functions appear as points on the curve of Fig. 3 when the unit of the ordinate is $2\frac{AT}{T_B}$.

In Fig. 6 the numerical values of the functions are shown plotted to a logarithmic scale.



$$|F(nxf_0)| = 2\frac{AT}{T_B}\frac{1}{\pi Tnf_0}\left|\frac{\sin \pi Tnf_0}{\pi Tnf_0} - \cos \pi Tnf_0\right| \quad \text{for } \tau = 0$$

$$|F(nxf_0)| = 2\frac{AT}{T_B}\frac{1}{\pi Tnf_0}\frac{T}{2\tau}\left|\frac{\sin \pi T(1-\frac{2\tau}{T})\,nf_0}{\pi T(1-\frac{2\tau}{T})\,nf_0} - \frac{\sin \pi Tnf_0}{\pi Tnf_0}\right| \quad \text{for } \tau > 0$$

Fig. 6. Logarithmic presentation of frequency spectrum for N-shaped pulses (sonic booms).

The Tape-Recorder

The tape-recorder used must offer certain facilities.

1. It has to be FM modulated in order to record and reproduce very low frequencies and DC without distortion or attenuation.
2. It must facilitate the use of closed tape loops. As seen in Appendix B it is of importance to use short tape loops.
3. The dynamic range of the tape-recorder must be as large as possible. (Modern instrumentation tape-recorders have dynamic ranges from 40–50 dB. The recorder used here has a dynamic range of 50 dB).

The dynamic range is measured from a maximum recording level. If the signal $f(t)$ is recorded at optimum that is the signal amplitude A equals the maximum level, the lowest level for measurement will be -50 dB rel. A.

4. The tape-recorder should have two channels. One of the channels can be used to record a signal where the lower frequencies are filtered away. The remaining signal will have a lower amplitude than the original signal (only if $\tau > 0$). Before recording, the signal is amplified to the same level as the original signal. This amplification raises the remaining frequencies to a higher level which allows analysis to higher frequencies. A combination of this recording and the recording of the original signal on the other channel gives an increase in the effective dynamic range. (See Fig. 6 and Appendix C.)
5. The tape-recorder must be able to run at different speeds in order to make possible a choice of $\frac{T}{T_B}$ when the signals are transfered to the tape loop. (See Appendix B.)
6. For practical measurements it was necessary to construct and apply an electronic gate to eliminate noise from tape splicings and other unwanted signals on the tape loop. A description of this gate is given in Appendix D.

Measuring Objects

The measurements described in this paper are done on tape-recordings of a number of sonic booms, with lengths $T \approx 120$ msec and risetimes $\tau \approx 4$ msec. In the following no reference is made to actual pressure amplitudes. Only the electrical amplitude of the signal is mentioned and frequency analysis is done with reference to this value.

Results

A number of different measuring set-ups were tried. Two of these are shown in Figs. 7 and 8.

In Fig. 7 a set-up is shown using a narrow band slave filter type 2020 which makes it possible to measure each of the frequencies in the line spectrum of

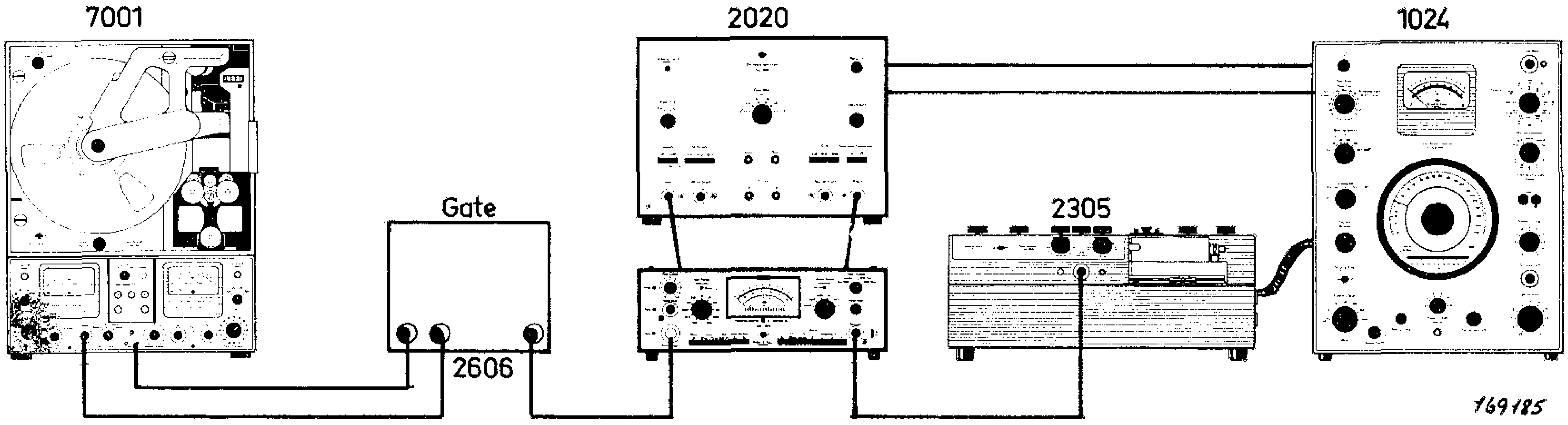


Fig. 7. Measuring set up employing heterodyne slave filter.

the periodic signal from the tape-recorder. By adjusting the "Frequency increment" knob of the Beat frequency oscillator type 1024, the analysis can be done from under 4.2 Hz. In Figs. 9, 10, 11, 12, 14 and 15 results obtained with this set up are shown.

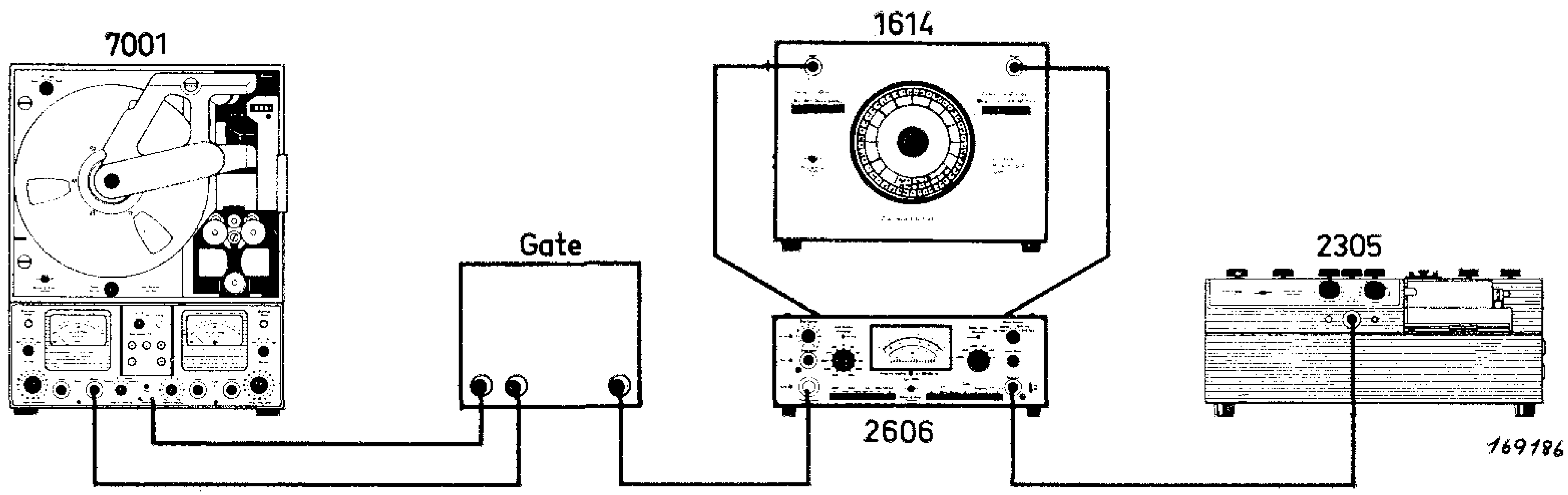


Fig. 8. Measuring set up employing 1/3 octave filter.

In Fig. 8 a measuring set up is shown using a 1/3 octave filter type 1614 which can analyze frequencies from 2 Hz. In Figs. 16 and 17 results obtained with this set up are shown. The measuring set ups were tested by analysis of rectangular and triangular pulses on tape recordings and the results were in good agreement with calculated values. In Figs. 9 and 10 are shown the analysis of two rectangular pulses of different lengths. It can be seen that the maxima and minima are distributed at intervals of $\frac{1}{T}$ on the frequency scale.

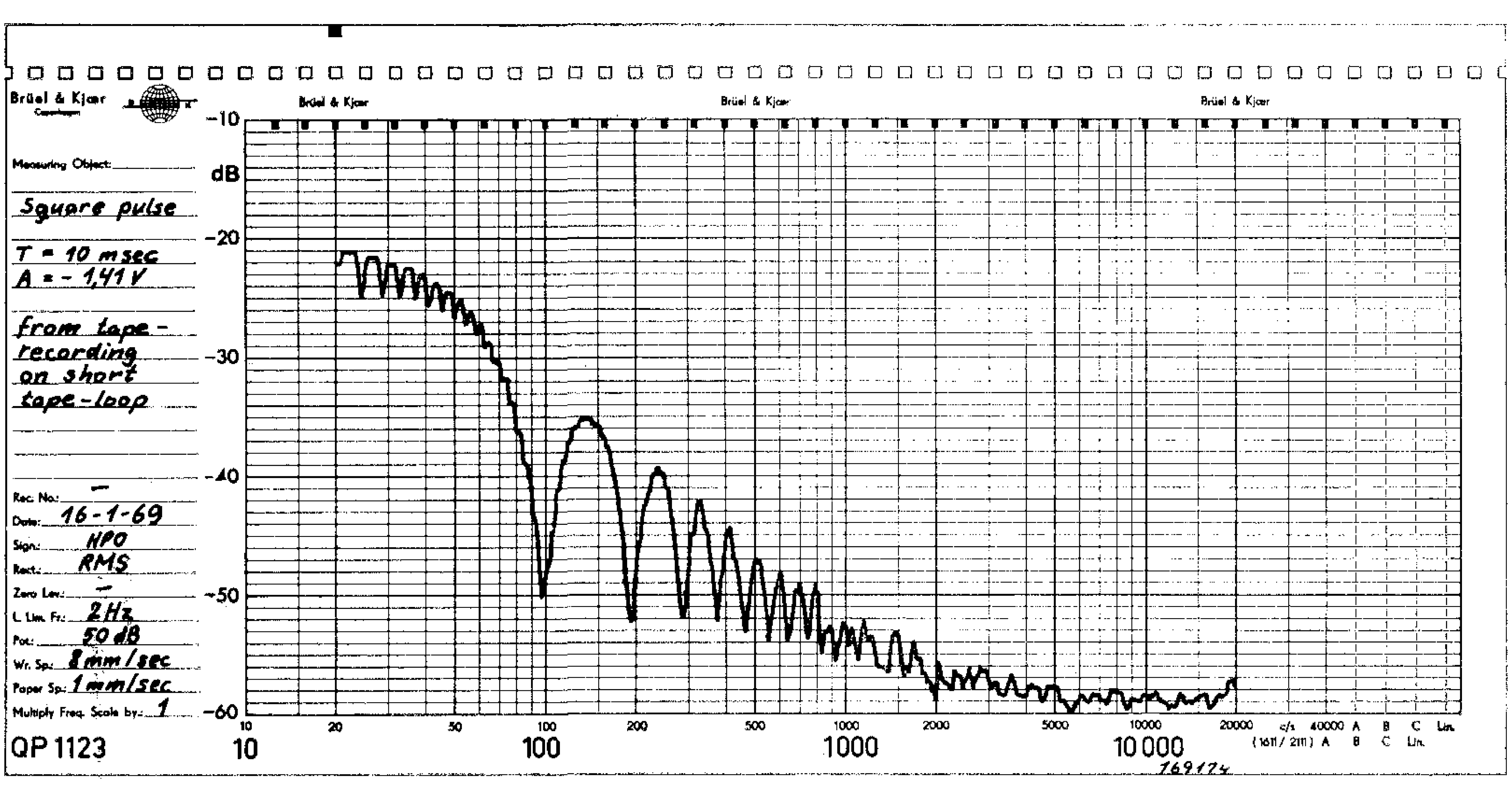


Fig. 9. Frequency spectrum recording of 10 msec. rectangular pulse.

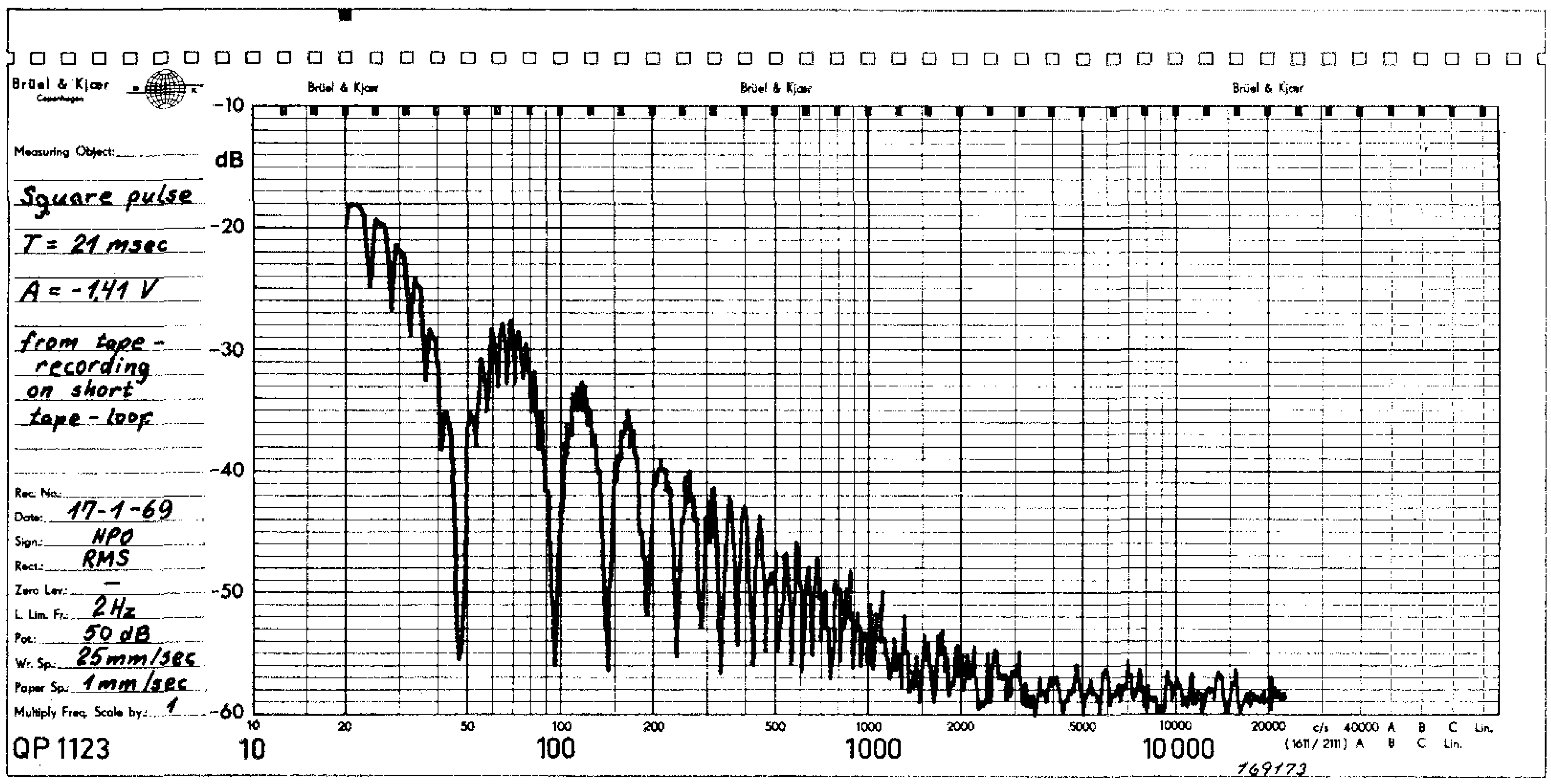


Fig. 10. Frequency spectrum recording of 21 msec. rectangular pulse.

In Fig. 11 one of these pulses analyzed from 0 Hz–by adjustment of frequency increment is shown. Fig. 12 is part of a similar analysis of the same pulse using another ratio between paperlength of the level recorder (type 2305) and the sweep length of the BFO (type 1024). In this way a better measurement of each frequency can be made and so the recorded form of the curve is better.

Fig. 13 shows two different sonic booms. Figs. 14, 15, 16 and 17 show measured results from analysis of the sonic boom in Fig. 13a with the two measuring set ups, and with two different lengths of pulse on the tape loop.

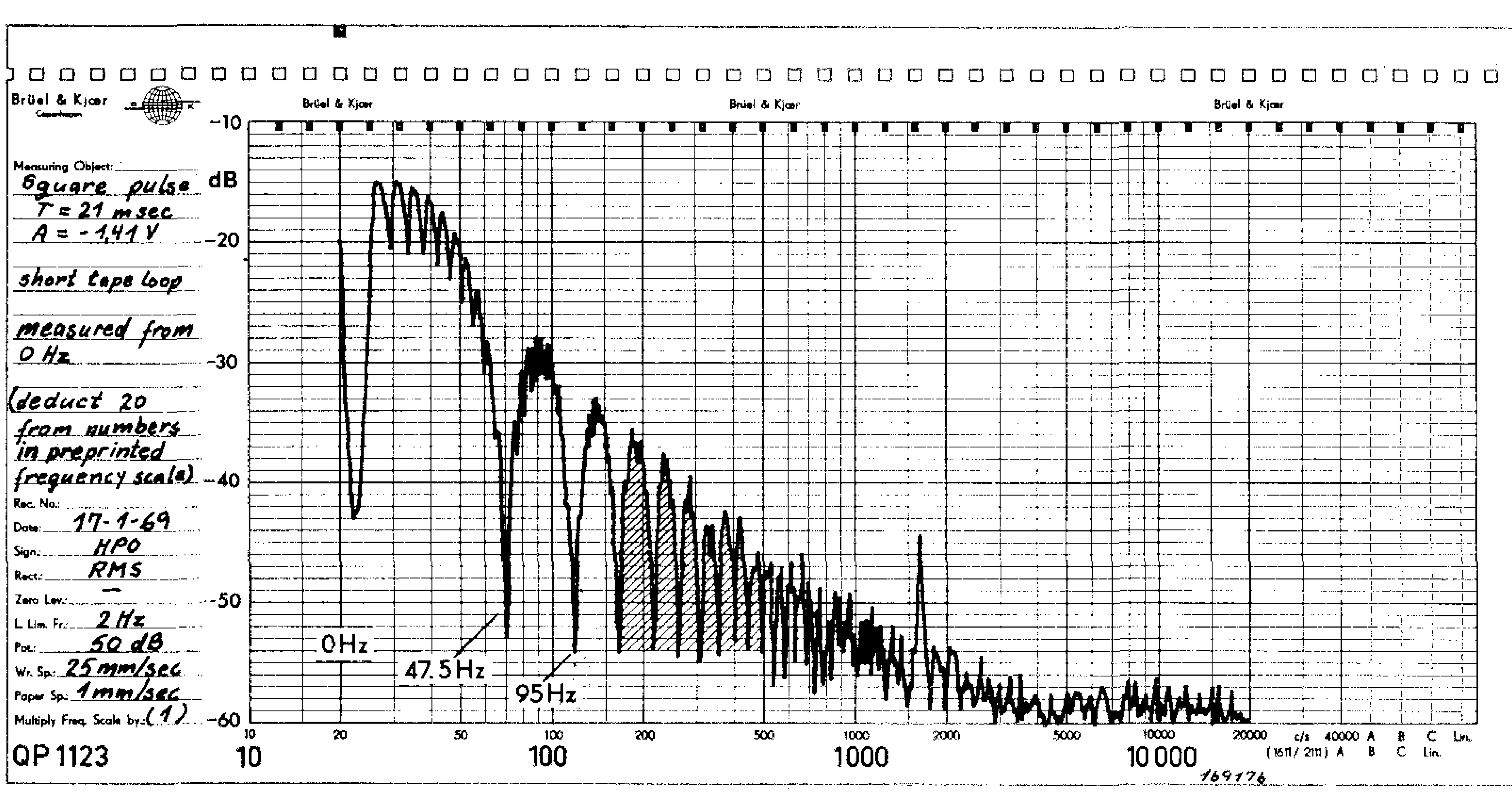


Fig. 11. Frequency spectrum recording of 21 msec. rectangular pulse recorded from 0 Hz. (Note shaded "hills").

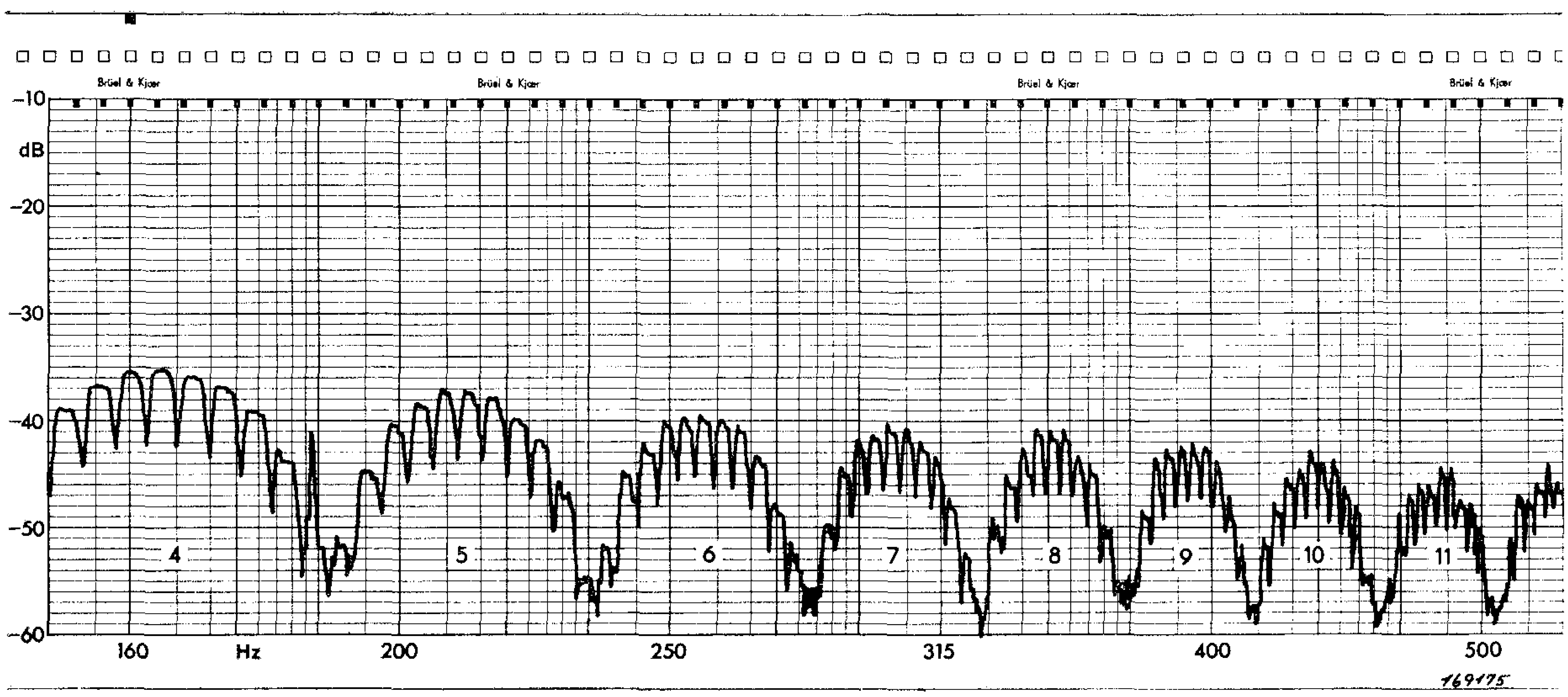


Fig. 12. Part of frequency spectrum recording of 21 msec. rectangular pulse using a greater paperlength to frequency ratio. (Compare with shaded "hills" in Fig. 11).

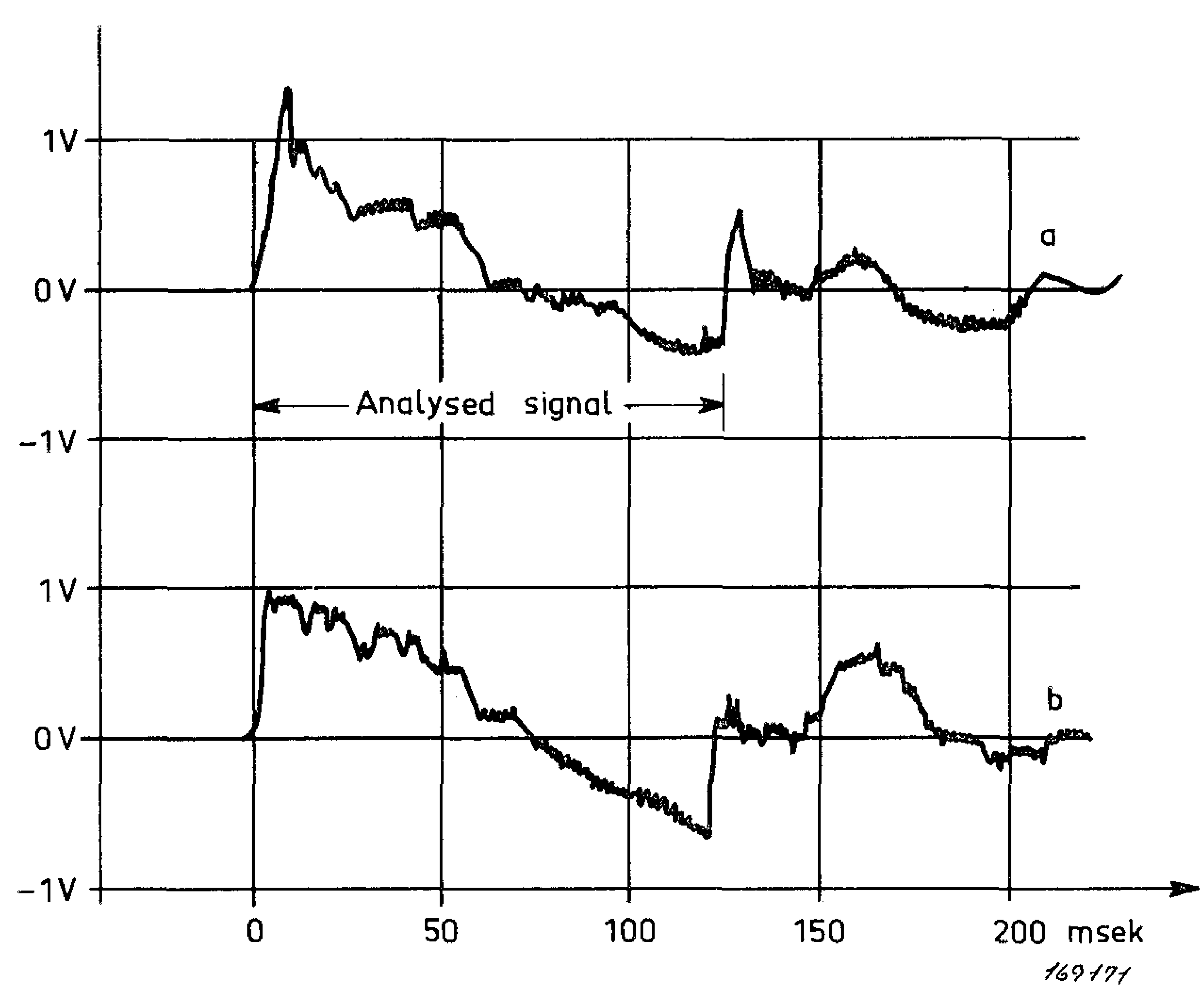


Fig. 13. Sonic booms. The upper one is used for the analysis and recordings in this paper.

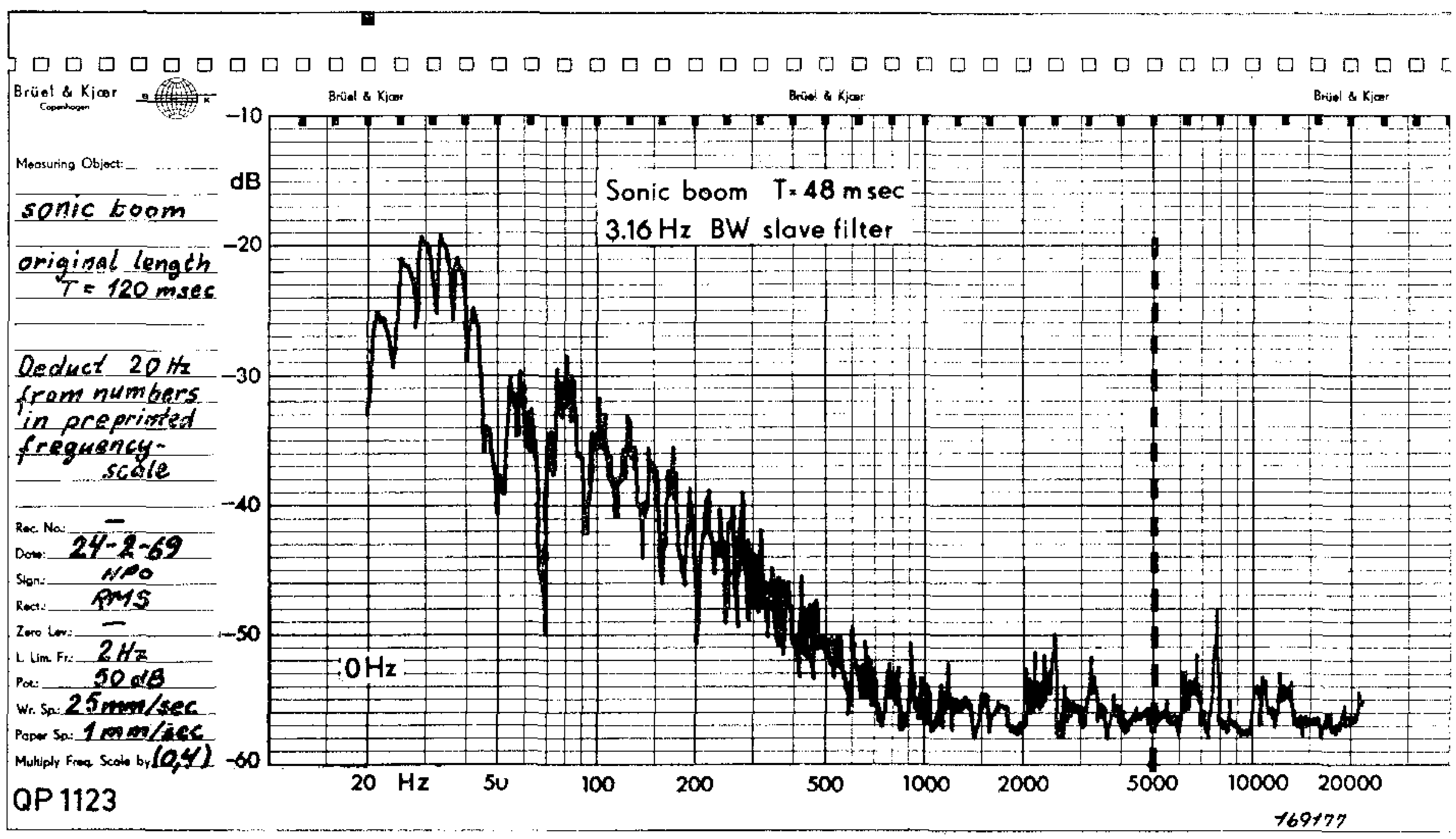


Fig. 14. Frequency spectrum recording of sonic boom for T = 48 msec.

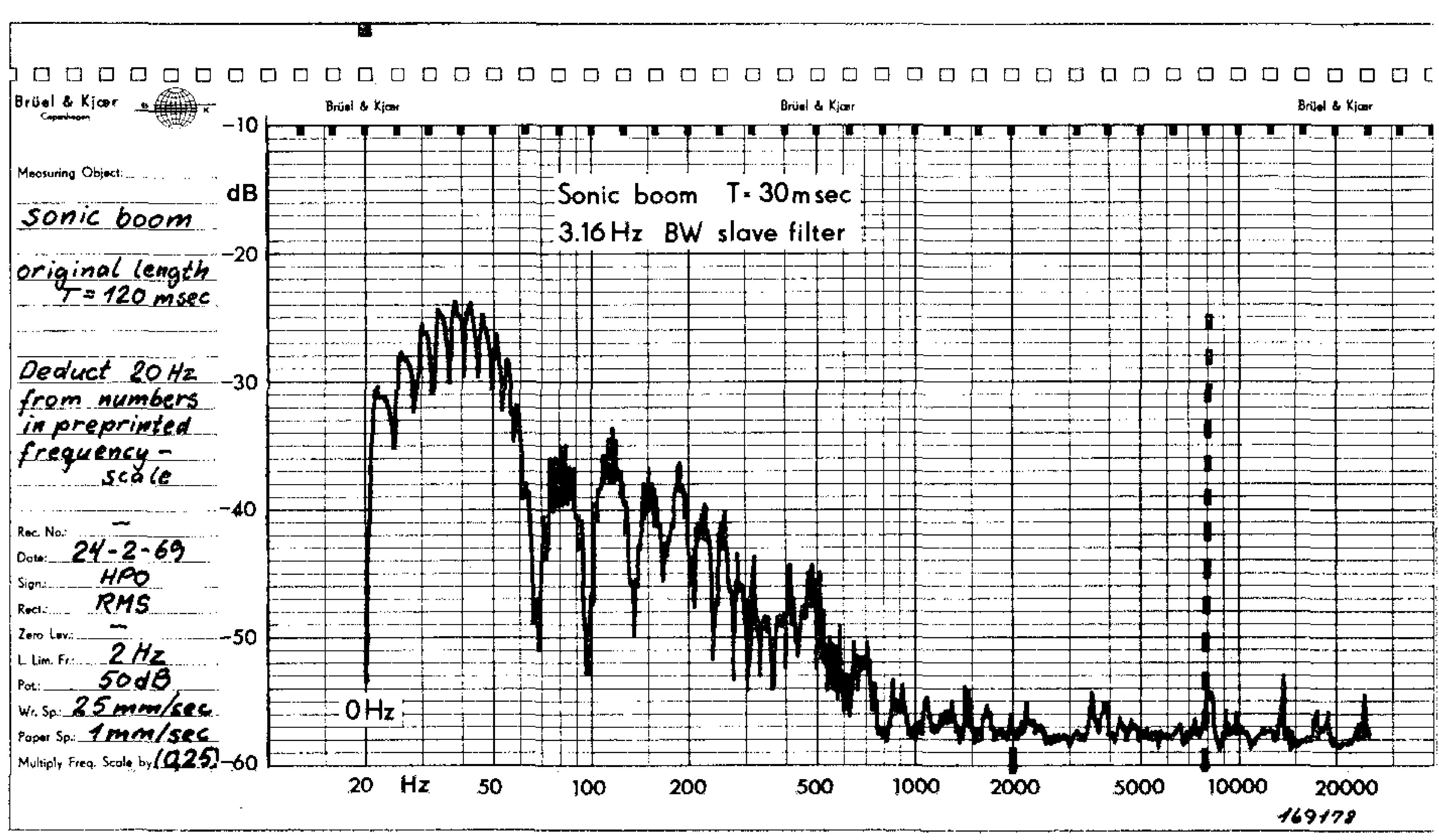


Fig. 15. Frequency spectrum recording of sonic boom for T = 30 msec.

By comparing Figs. 14 and 15 with Figs. 16 and 17 respectively it is seen that the narrow band filter type 2020 gives a very detailed picture of the frequency function but that the signal at higher frequencies disappears in noise (at −50 dB). The −1/3 octave filter type 1614 gives information on the separate frequencies up to 25 Hz but gives at higher frequencies the square sum of an increasing number of frequencies.

As the frequency increases the detail of the curve gets worse but the signal/noise ratio becomes better than that of the type 2020.

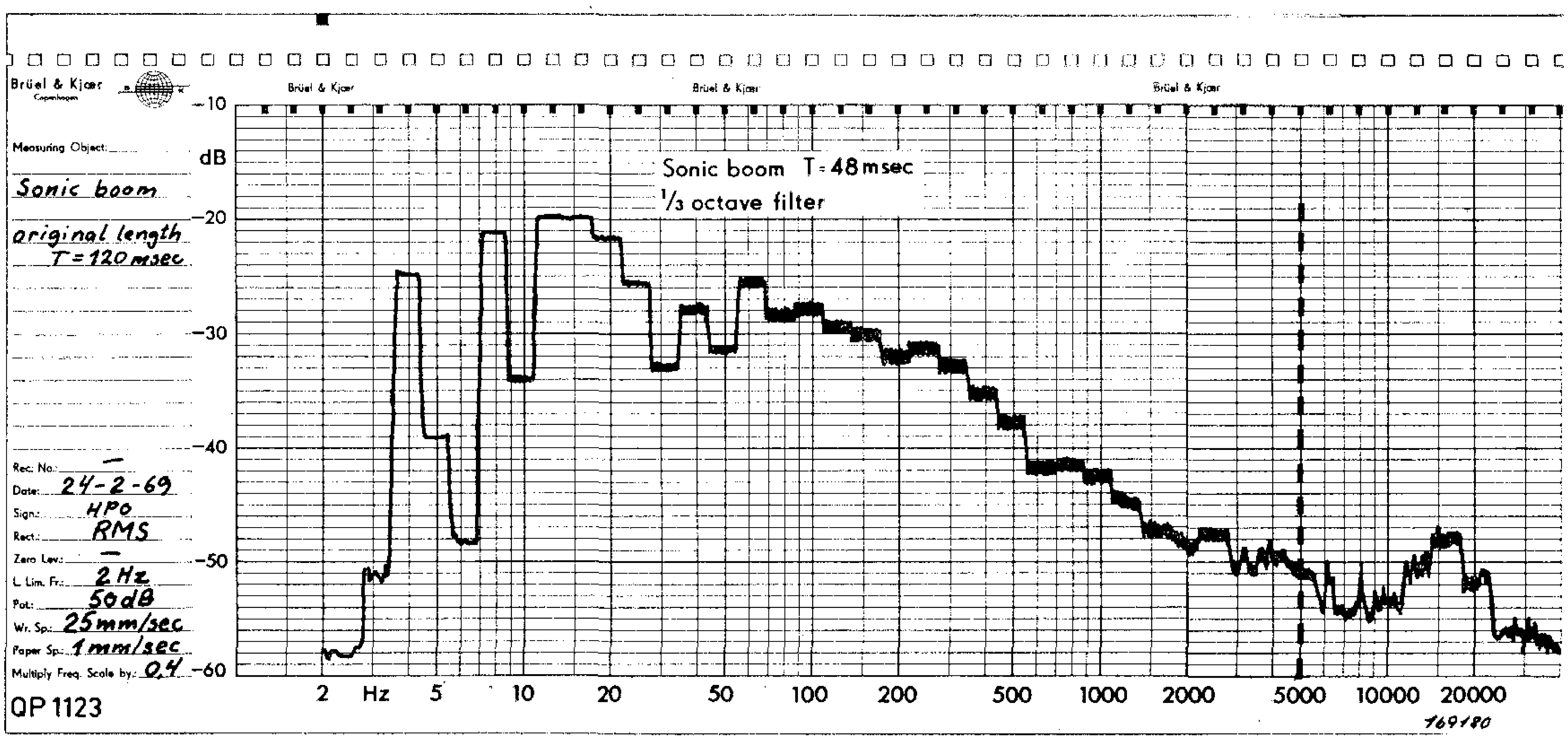


Fig. 16. Frequency spectrum recording of sonic boom for T = 48 msec. using 1/3 octave filter. (Compare with Fig. 14).

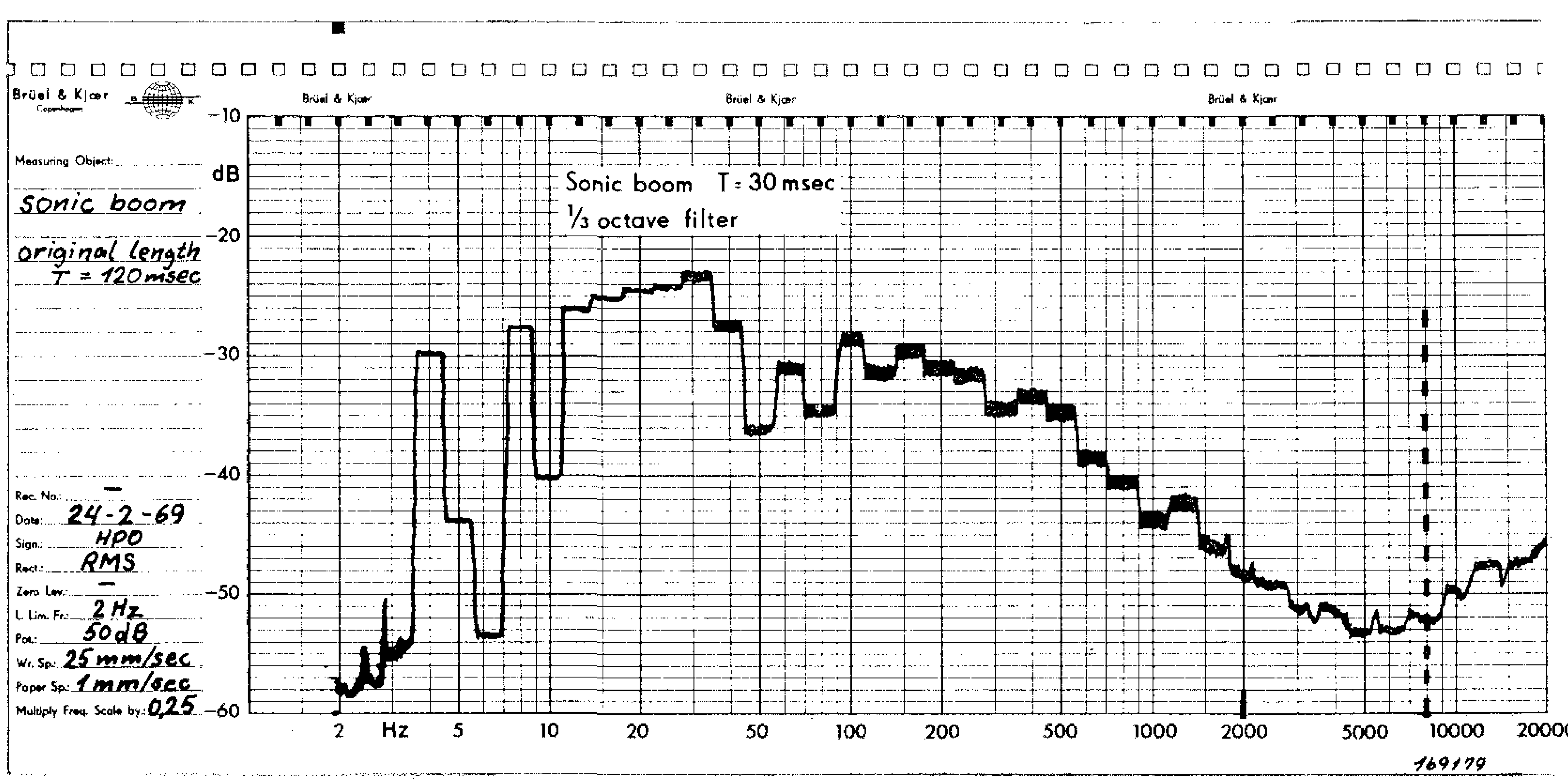


Fig. 17. Frequency spectrum recording of sonic boom for T = 30 msec. using 1/3 octave filter. (Compare with Fig. 15).

Conclusion

The measurements made have shown that an FM modulated tape-recorder set up with the necessary analyzing instruments is a powerful tool for frequency analysis of single pulses. It is, however, important to be aware of the limitations set by the measuring instruments and to establish a procedure of measurement which takes into account the measuring instruments and the general form of the signal as well as the particular parameters to be investigated during the analysis.

Appendices

A. Fourier Integral Transformation

The frequency content of single pulses is found by applying the Fourier Integral.

$$F(f) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\, dt \tag{1}$$

This integral is calculated for:

1. A rectangular single pulse (see Fig. 1)

$$f(t) = \begin{cases} A \text{ for } -\dfrac{T}{2} < t < \dfrac{T}{2} \\ 0 \text{ elsewhere} \end{cases}$$

$$F(f) = \int_{-\frac{T}{2}}^{\frac{T}{2}} A\, e^{-j2\pi ft}\, dt$$

$$F(f) = AT \frac{\sin \pi Tf}{\pi Tf} \tag{1.1}$$

2. A triangular single pulse (see Fig. 2)

$$f(t) = \begin{cases} A\,(1 + \dfrac{t}{T}) \text{ for } -T < t < 0 \\ A\,(1 - \dfrac{t}{T}) \text{ for } 0 < t < T \\ 0 \text{ elsewhere} \end{cases}$$

$$F(f) = \int_{-T}^{0} A\,(1 + \frac{t}{T})\, e^{-j2\pi ft}\, dt + \int_{0}^{T} A\,(1 - \frac{t}{T})\, e^{-j2\pi ft}\, dt$$

$$F(f) = AT \left(\frac{\sin \pi Tf}{\pi Tf}\right)^2 \tag{1.2}$$

3. An N-shaped single pulse (see Fig. 3)

$$f(t) = \begin{cases} A\left(\dfrac{t+\dfrac{T}{2}}{\tau}\right) & \text{for } -\dfrac{T}{2} < t < -\dfrac{T}{2}+\tau \\ -A\left(\dfrac{t}{\dfrac{T}{2}-\tau}\right) & \text{for } -\dfrac{T}{2}+\tau < t < \dfrac{T}{2}-\tau \\ A\left(\dfrac{t-\dfrac{T}{2}}{\tau}\right) & \text{for } \dfrac{T}{2}-\tau < t < \dfrac{T}{2} \\ 0 \text{ elsewhere} \end{cases}$$

$$F(f) = \int\limits_{-\frac{T}{2}}^{-\frac{T}{2}+\tau} A\left(\frac{t+\frac{T}{2}}{\tau}\right) e^{-j2\pi ft}\, dt - \int\limits_{-\frac{T}{2}+\tau}^{\frac{T}{2}-\tau} A\left(\frac{t}{\frac{T}{2}-\tau}\right) e^{-j2\pi ft}\, dt + \int\limits_{\frac{T}{2}-\tau}^{\frac{T}{2}} A\left(\frac{t-\frac{T}{2}}{\tau}\right) e^{-j2\pi ft}\, dt$$

$$F(f) = j\,\frac{A}{(\pi f)^2}\,\frac{1}{2\,\tau\,(1-\frac{2\,\tau}{T})}\left(\sin \pi T\,(1-\frac{2\,\tau}{T})\,f - (1-\frac{2\,\tau}{T})\sin \pi Tf\right)$$

$$F(f) = j\,AT\,\frac{1}{\pi Tf}\,\frac{T}{2\,\tau}\left(\frac{\sin \pi T\,(1-\frac{2\,\tau}{T})\,f}{\pi T\,(1-\frac{2\,\tau}{T})\,f} - \frac{\sin \pi Tf}{\pi Tf}\right) \tag{1.4}$$

The expression for $F(f)$ for $\tau = 0$ can be found:

$$F(f) = j\,\frac{A}{(\pi f)^2}\left(\frac{\sin \pi Tf \cos 2\,\pi\tau f - \cos \pi Tf \sin\, 2\,\pi\tau f - (1-\frac{2\,\tau}{T})\sin \pi Tf}{2\,\tau\,(1-\frac{2\,\tau}{T})}\right)$$

$$F(f) = j\,\frac{A}{\pi f}\left(\frac{\sin \pi Tf}{\pi Tf\,(1-\frac{2\,\tau}{T})}\left[\frac{T}{2\,\tau}\cos 2\,\pi\tau f - \frac{T}{2\,\tau} + 1\right] - \cos \pi Tf\, \frac{\sin\, 2\,\pi\tau f}{2\,\pi\tau f\,(1-\frac{2\,\tau}{T})}\right)$$

for $\tau \to 0$ we have

$$F(f) = j\,AT\,\frac{1}{\pi Tf}\left(\frac{\sin \pi Tf}{\pi Tf} - \cos \pi Tf\right) \tag{1.3}$$

B. The Tape-Recorder

Transformation of Frequencies

When a recorded signal is played back at different speeds the frequency content of the signal will be changed i.e.

$$\frac{f_1}{v_1} = \frac{f_2}{v_2} = \frac{f_3}{v_3}$$

The duration of the signal will also be changed i.e.

$$T_1 \times v_1 = T_2 \times v_2 = T_3 \times v_3.$$

Thus the frequencies or the duration of the signal which are best suited to the following instruments can be selected.

Actually the speed of playback is preferred to be constant for reasons explained lated in this chapter, so the change of signal duration and frequencies is done when transferring the signal to the closed tape loops. The transfer is done by recording on one tape recorder the signal played back on another, the two tape recorders running at various speeds. In this procedure one should be conscious of the upper frequency limits of the tape recorders.

In the experiments described here B & K tape-recorders type 7001 were used which provide the following tape speeds and bandwidths.

V	BW
1.5 in/sec ≈ 3.81 cm/sec	dc – 500 Hz
6 in/sec ≈ 15.25 cm/sec	dc – 2000 Hz
15 in/sec ≈ 38.1 cm/sec	dc – 5000 Hz
60 in/sec ≈ 152.5 cm/sec	dc – 20000 Hz

The tape-recorder has a dynamic range of 50 dB with a maximum recorded level of 1.41 V peak or 1 V RMS sine.

Selection of the ratio $\frac{T}{T_B}$

Various factors have to be taken into consideration before selecting the ratio between the pulse duration on the tape loop and its repetition time.

1. *Signal/noise ratio.*
   From the expressions for $F(n \times f_0)$ on pages 6 and 7 it can be seen that the amplitudes of the frequencies are proportional to $\frac{T}{T_B}$ so that the

signal/noise ratio is improved by raising the ratio $\frac{T}{T_B}$.

2. *The curve form of the frequency spectrum.*
From Figs. 1, 2 and 3 it is seen that the functions are distributed in "hills" of approximate width $\frac{1}{T}$. The more frequencies there are in each "hill" the better the curve form is defined. The distance between frequencies is $\frac{1}{T_B}$.
The number of frequencies in each "hill" is $\frac{T_B}{T}$ and hence the resolution of the curve form is better the smaller $\frac{T}{T_B}$ is.

3. *The crest factor of the signal.*
The measuring instrument used to measure and analyze the periodic signal from the tape-recorder sets a limit to the crest factor of the signal. The B & K instruments used in this experiment accept crest factors up to 5 ($f_{cmax} = 5$). The crest factor of a signal is calculated from

$$f_c = \frac{f(t)_{peak}}{f(t)_{RMS}}$$

Where $f(t)_{peak} = A$ for all signals used here

and $f(t)_{RMS} = \sqrt{\frac{1}{T_B}\int_0^{T_B} [f(t)]^2\, dt}$

The rectangular pulse in Fig. 1 has

$$f(t)_{RMS} = A\sqrt{\frac{T}{T_B}}$$

and

$$\frac{T}{T_B}\min = \frac{1}{(f_c \max)^2} = \frac{1}{25}$$

The triangular pulse in Fig. 2 has

$$f(t)_{RMS} = A\sqrt{\frac{2T}{3T_B}}$$

and

$$\frac{T}{T_B}\min = \frac{3}{2(f_c \max)^2} \approx \frac{1}{17}$$

The N-shaped pulse in Fig. 3 has

$$f(t)_{RMS} = A \sqrt{\frac{T}{3\,T_B}} \text{ independent of the value of } \tau$$

and

$$\frac{T}{T_B} \text{ min} = \frac{3}{(f_c \text{ max})^2} \approx \frac{1}{8}$$

4. *Other factors affecting measurements.*
   Recording instruments which can rectify and show AC values (here $F(n \times f_o)$) can normally not handle the signals correctly at repetition frequencies below approximately 2 Hz. By using a special arrangement on the B & K Type 7001 it has been possible to adapt a tape loop length of 0.362 m (Normal length is 2.4 m or more i.e. $T_B = 1.57$ sec. or more). This short tape loop provides at the highest tape speed a repetition time T_B of 0.238 sec. and a repetition frequency of $f_o = 4.2$ Hz which is higher than the minimum 2 Hz. The low frequency limit of 2 Hz could be lowered by changing the integration time-constant of the recording instruments but this would increase the analyzing time. Also it is important that the repetition frequency f_o (here 4.2 Hz) be > 3.16 Hz which is the narrowest filter bandwidth of the slave filter type 2020. This allows the measurement of single frequencies in the whole line spectrum.

5. *Conclusion.*
   As seen from the preceeding remarks, the limitations of the different instruments require that the rate $\frac{T}{T_B}$, as well as the absolute value of T_B, and the tape speed of the first recording of the signal, be selected with careful consideration to the kind and the size of the parameters on which knowledge is wanted.
   For a "sonic boom" an ISO proposal suggests a minimum bandwidth of 0.1–5000 Hz for recordings on tape-recorders. For the B & K type 7001 this would demand an initial recording speed of 15 in/sec. (Also the ISO suggests another recording from 100–5000 Hz on another channel in order to improve the signal/noise ratio of higher frequencies.)
   Now T_B can be 238 msec. or 1.57 sec. or more. The choice depends on the possibility of recording signals with repetition frequencies of 0.6 Hz or lower. Also the resulting analyzing time must be considered. The duration of the original signal also has some influence on the final choice of T_B.
   The ratio $\frac{T}{T_B}$ is made as small as possible– limited only by the crest factor– when the curve form in the low frequency region is investigated. Here $\frac{T}{T_B}$ min $\approx \frac{1}{8}$ (8 frequencies/hill).

In order to get a better signal/noise ratio $\frac{T}{T_B}$ is made as large as possible when investigating higher frequencies. If at least one frequency in each "hill" must have approximately the amplitude of the "hill" the limit must be $\frac{T}{T_B}$ max $\approx \frac{1}{3}$.

It should be mentioned that at $T_B = 1.57$ sec. there will always be more than one frequency at a time in each filter in commercially available analyzers. In this way a long repetition time tends to give only an approximate picture of the curve form even when $\frac{T}{T_B}$ min is used.

C. Tape Recording Sonic Booms

Fig. 6 shows that the frequency spectrum of a sonic boom falls off rapidly with increasing frequency. At a frequency decided by the ratio $\frac{T}{\tau}$ an initial fall of 20 dB/decade changes to 40 dB/decade.

If a choice is made of $\frac{T}{T_B} = 3$ (3 frequencies/hill in the recorded results) it can be seen that the recorded level A on the tape-recorder must be $20 \log \frac{T_B}{2T}$ $= 3.5$ dB above the shown 0 dB level.

The tape recorder has a dynamic range of 50 dB if the signal is under optimum conditions, (i.e. at 0 dB on the recording meter). This range is however shortened when lower recording levels are used. This means that no results can be expected from levels below approximately -46 dB when recording level A is used as in Fig. 6. This level corresponds to frequencies between $\frac{25}{T}$ and $\frac{100}{T}$ depending on $\frac{T}{\tau}$. With a 1/3 octave filter, however, the possible frequency range of measurement will be extended somewhat (see Appendix E). If higher frequencies are to be analyzed the signal must be recorded completely on one of the tape-recorder channels, and with the lower frequencies filtered away on another channel. The signal on the second channel can be amplified more than the complete signal without exceeding the recording limit and thus the analysis can be made to higher frequencies.

D. Frequency Scanning

The speed at which the frequency range can be scanned is limited by the filter and by the level recorder. The filter must have time to rise to full amplitude for each frequency and the pen of the level recorder must have time to reach the correct level before the frequency leaves the filter.

E. Evaluation

By applying measuring instruments the original signal is changed. The results obtained must therefore be evaluated before they give information about the properties of the original signal.

1. The original signal is a pressure wave $P(t)$.
2. The signal is changed by a transducer (microphone) to an electrical signal $f(t) = M(f) \times P(t)$.
3. The signal is made periodic by using a closed tape loop on the tape-recorder.
   The continuous frequency spectrum $F(f)$ is changed to a line spectrum:
   $$C_n = F(n \times f_o) = \frac{2}{T_B} F(f) \text{ for } f = n \times f_o$$
4. The frequency content of the signal is changed by changing the signal duration from T to T_1.
5. The signal is applied to a filter through which only part of the signal passes. If more than one frequency passes the filter at a time, the output is the sum of these frequencies.
6. The signal is rectified. Here a true RMS rectifier is used which gives:
   $$F_{rect}\,(n \times f_o) = \sqrt{\Sigma \,|\, F_{filter}\,(n \times f_o)\,|^2}$$
7. The signal might be amplified or attenuated on its way through the measuring system.
8. The recording of the result is in dB on a level-recorder supplied with a logarithmic potentiometer.

A combination of the paper record from the level recorder and the information in the above 8 points gives information on the spectrum of the original signal.
As an example the results in Figs. 14 and 16 are discussed here:
The result in Fig. 14 is obtained by frequency analysis of the pulse in Fig. 13a. The analysis has been made on the indicated part of the pulse, the sonic boom. The reflection following the sonic boom has been eliminated by an electronic gate.
The signal has been made periodic with a repetition time $T_B = 238$ msec.

$$C_n = \frac{2}{T_B} F(f) \approx 8.2\, F(f)$$

The frequency content of the signal has been changed by changing the pulse duration from

$$T \approx 120 \text{ msec to } T_1 \approx 48 \text{ msec.}$$

$$f_o = \frac{T}{T_1} f = 2.5\, f$$

The amplitudes of the separate frequencies have been changed to:

$$C_{n1} = \frac{T_1}{T}\, C_n = \frac{1}{2.5}\, C_n$$

The filter bandwidth is 3.16 Hz so only one frequency passes at a time. Each frequency is rectified and indicated by the level recorder in dB rel. 1 V RMS sine. The frequency content of the original signal is found by:

1. Correction of the preprinted frequency scale. After subtracting the 20 Hz frequency increment from all numbers of the scale they are divided by 2.5. For the lower frequencies the correct frequency value can also be computed by counting the number of frequencies from 0 Hz, multiplying the number by 4.2 and dividing it by 2.5.
2. Drawing a line between the separate amplitudes.
3. Then changing the amplitude scale from dB rel. 1 V RMS to dB rel. 1 V sec. This is done by adding the following value to the used scale:

$$20 \log \left(\frac{T_B}{2} \times \frac{T}{T_1}\right) = 20 \log \frac{0.238 \times 2.5}{2} = -10.5 \text{ dB}$$

$$\left(F(f) = \frac{T_B}{2} C_n = \frac{T_B}{2} \frac{T}{T_1} C_{n1}\right)$$

The result in Fig. 16 is in two ways different from that of Fig. 14.

1. The frequency scale here is only divided by 2.5.
2. To the amplitude scale is added −10.5 dB but as a 1/3 octave filter has been used, at higher frequencies more than one frequency at a time has been passed by the filter. Comparison of Figs. 14 and 16 shows that the recorded values are alike to approximately 25 Hz, then the bandwidth of the 1/3 octave filter becomes wider than the distance between separate frequencies. From this frequency on, the recorded signal in Fig. 16 is raised approximately 3 dB/octave compared to that of Fig. 14. (3 dB/octave is the theoretical value expected).

   From the recording in Fig. 16 the curve form of the frequency spectrum cannot be seen but the signal level at higher frequencies is kept well above the noise level of the tape recorder and so in this respect this recording is better than that in Fig. 14.

F. Electronic Gate

When using closed tape loops on a tape-recorder the noise pulse appearing at tape splices often disturbs measurements particularly at high frequencies. It was decided to construct a gate which could eliminate the noise from the necessary tape splice. In addition the circuit was found to be just as effective in eliminating other unwanted signals on the tape loop.

In Fig. 18 is shown a block diagram of the gate and below it are the signals appearing at various points of the gate.

a. A triggering signal from the tape recorders monitor output is led to the triggering circuit.

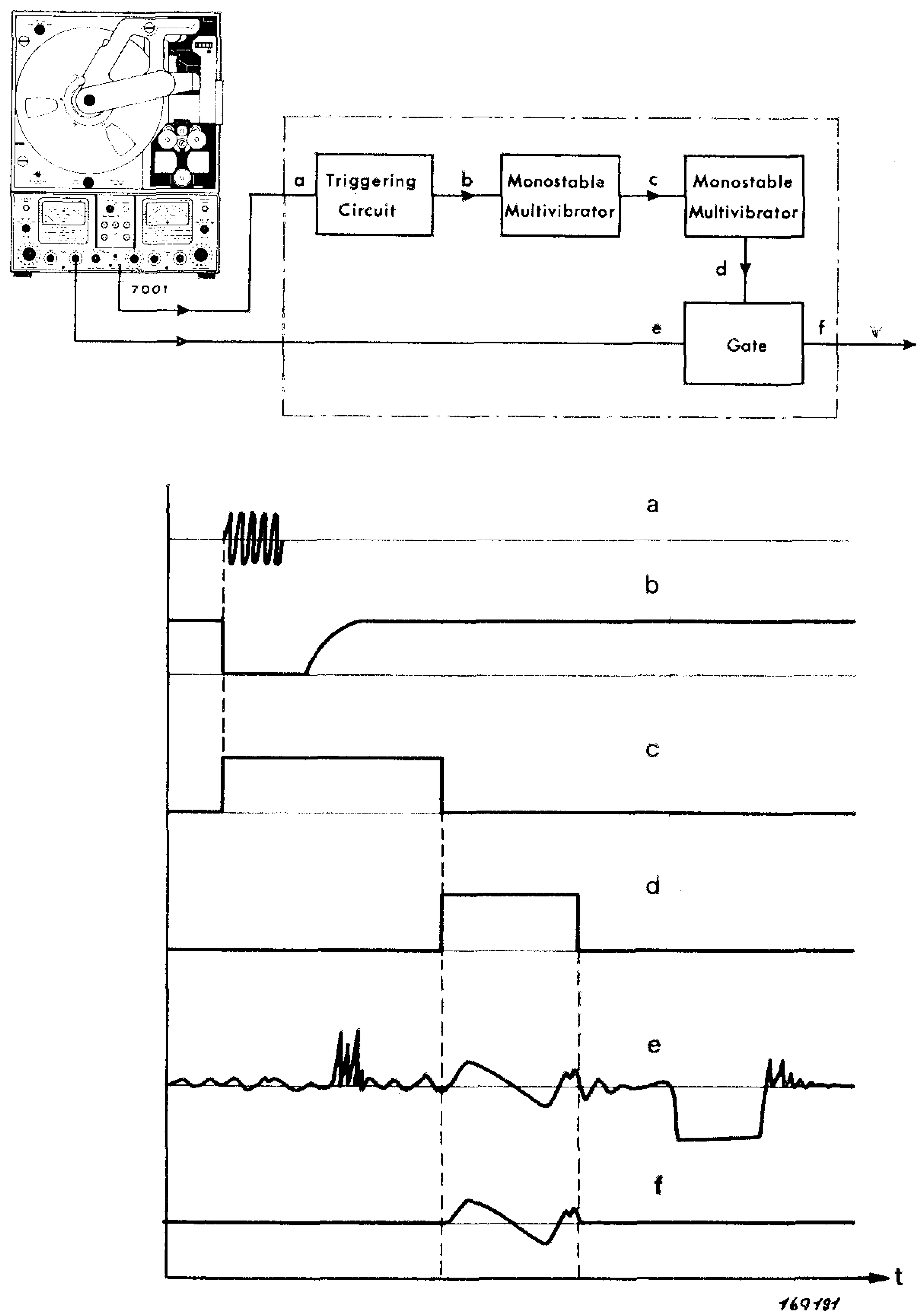


Fig. 18. Block diagram and functional sequence of electronic gate.

b. A signal from the triggering circuit triggers a monostable multivibrator.
c. When triggered the multivibrator's output level changes to a positive value. This positive output level is held for a certain time (preadjusted timelag) then the output level goes back to 0 V whereby the next multivibrator is triggered.
d. This multivibrator now changes its output level and it keeps this level for a certain time. The output signal is led to the gate and allows this to open.
e. A signal from one of the measuring channels of the tape recorder is led to the gate input.
f. The gate output is held at a level of 0 V until the second multivibrator allows the input signal to pass through to the output. When the multivibrator output goes back to its initial level the signal on the gate output

again goes to 0 V. By adjusting the timelags of the two multivibrators the gate can allow the signal from any part of the tape loop to pass for analysis while signals on all other parts of the tape loop are eliminated.

Triggering pulse on the tape-recorder voice channel.
When placing the triggering pulse on the tape loop it is advisable to let the tape loop run at 6 in/sec. Then it is possible to place the pulse with some accuracy on the tape loop and also the limited bandwidth of the voice channel is taken into account. The triggering pulse is applied by using the built in marking knob.

The practical solution.
The circuits used are the result of an experimental set up which works perfectly, but at present some attempt to optimize the circuits is being done. When this work is finished diagrams of the circuits will be available on request.

Power Supply
For practical reasons the B & K type 2805 was selected. The type 2805 supplies a regulated ± 14 V. A fixed voltage power supply eliminates time consuming voltage adjustments when the circuits are set up.

Triggering Circuit (Fig. 19)
The triggering circuit consists of a Schmidt-trigger followed by a rectifier and an output circuit.

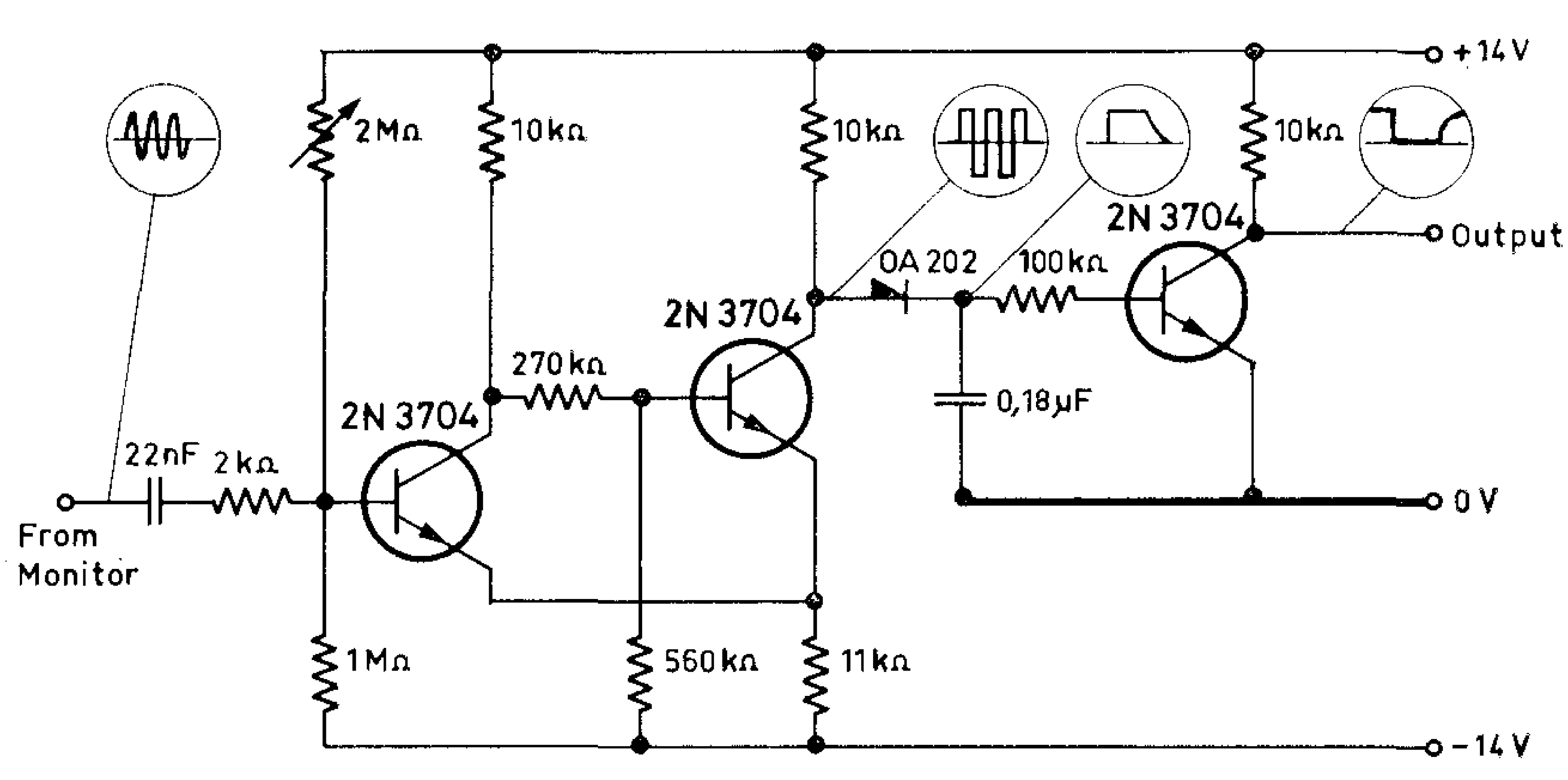


Fig. 19. Triggering circuit.

Monostable Multivibrator (Fig. 20)

The circuit consists of a bistable multivibrator (flip-flop) and a triggering circuit which resets the flip-flop after it has been triggered by an external signal:

When a negative step is applied to the triggering input the multivibrator changes output levels. Output 1 achieves a positive potential which is led over an RC chain to the emitter of a unijunction-transistor. When the condenser of the RC chain has been charged to the firing voltage of the unijunction transistor the condenser is discharged through the unijunction transistor whereby the multivibrator is reset. Output 1 can be used to trigger another multivibrator at the end of the preset time as a negative step is generated here at that time.

This circuit was chosen because of its better time measuring properties and the output signals have smaller fall times than an ordinary two transistor monostable multivibrator.

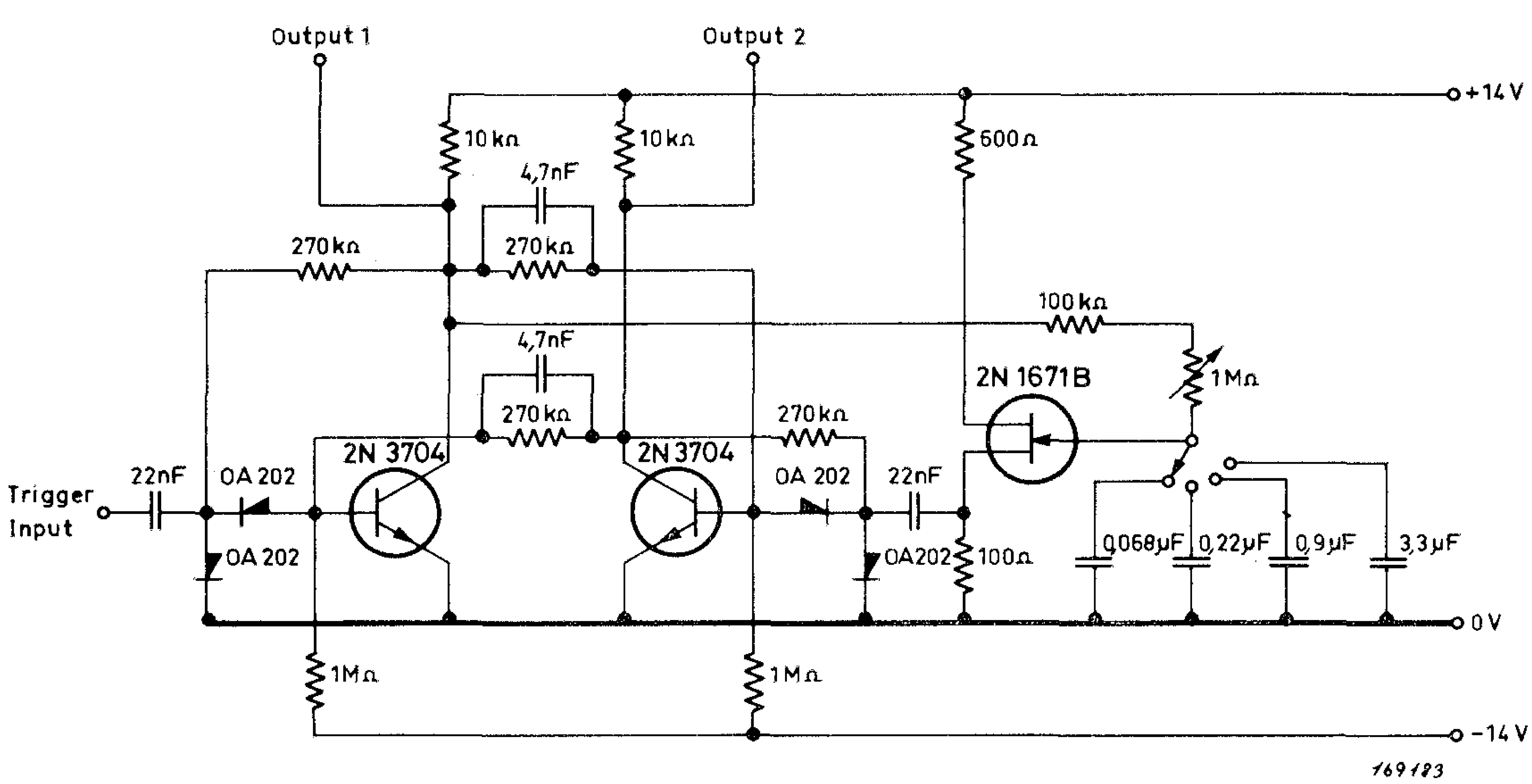


Fig. 20. Monostable multivibrator.

Gating Circuit (Fig. 21)

The gate consists of an amplifier, which amplifies 6 times, followed by an attenuator which attenuates the output signal to the input signal level. To lower the output impedance an emitter follower is used in the output circuit. When the potentials of outputs 1 and 2 of multivibrator 2 are respectively + 14 V and 0 V (when multivibrator 2 is triggered) transistors Q1 and Q2 in the gate are both in the "off" condition and the signal from the tape recorder can pass as described above.

When multivibrator 2 is reset the output levels will change. Transistors Q1 and Q2 will be saturated and will short-circuit point (1) to − 14 V and point (2) to + 14 V. This forms a voltage divider. By adjusting this, an output voltage of 0 V can be obtained. This output level is held until a new triggering pulse triggers the system and so lets a new tape-recorder signal pass.

The semiconductors used were:

Integrated linear amplifier, PHILCO	PA 7709
Transistor NPN, Texas Instruments	2N 3704
Transistor PNP, Texas Instruments	2N 3702
Unijunction-transistor, Thomson & Houston	2N 1671 B
Diode, Philips	OA 202
Zener diode 10 V, Silec	1N 708 A

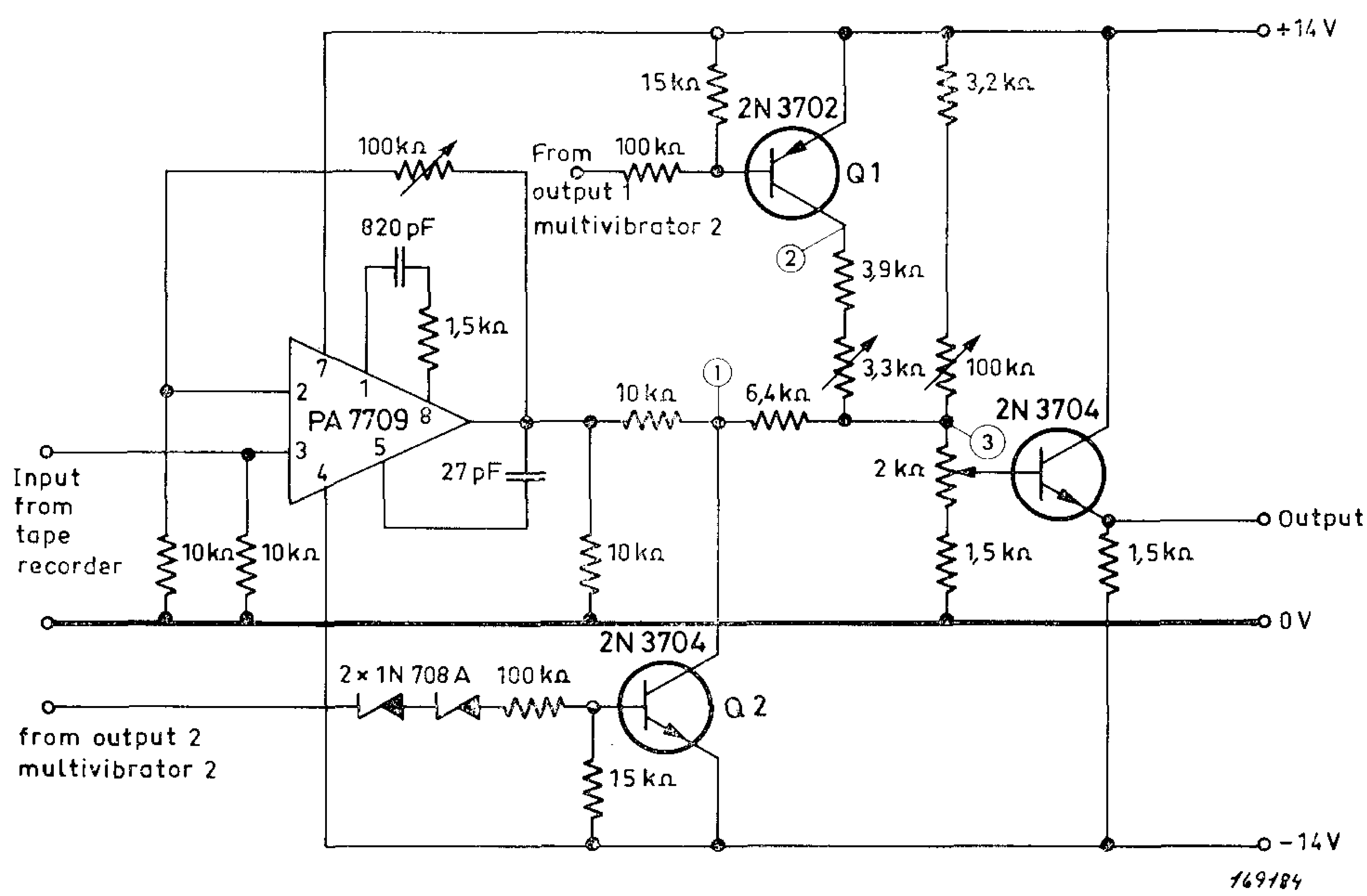


Fig. 21. Gating circuit.

News from the Factory

Filter Sets Types 1614/15

The new filter sets are a development of the Type 1612, incorporating many new features which give a generally improved performance.

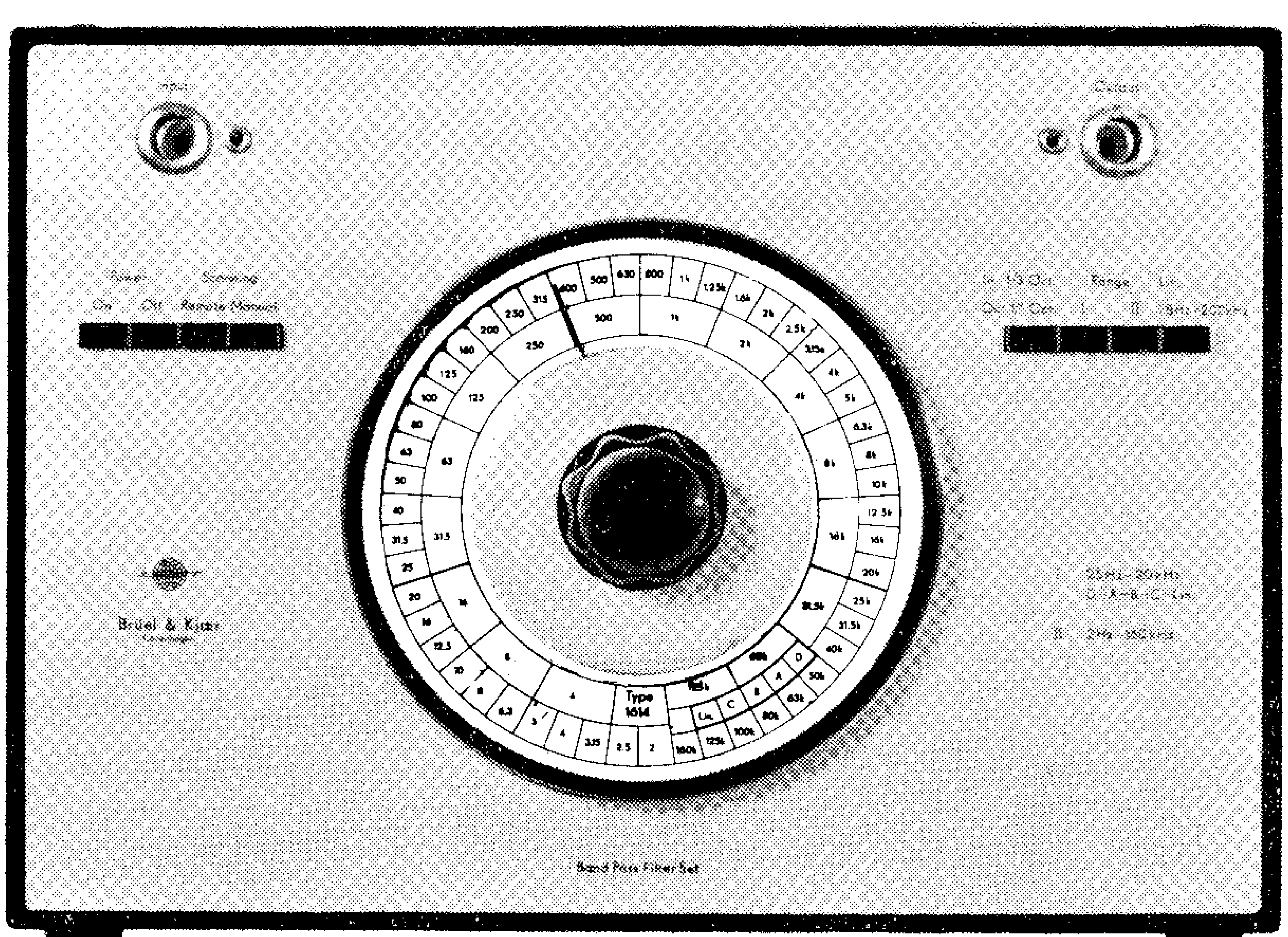
The selective range of frequencies of the Type 1614 is 2 Hz to 160 kHz (third octave centre frequencies). A secondary range from 25 Hz to 20 kHz can be selected for analysis of audio-frequency signals. The 1615 is a version of the 1614 containing the audio frequency range filters, 25 Hz to 20 kHz. The plug-in unit construction makes it a simple matter to convert the 1615 set to the range of the 1614.

Both Type 1614 and Type 1615 contain A, B, C, and D weighting networks and two linear ranges from 22.4 Hz to 22.4 kHz, and from 1.8 Hz to 200 kHz. Filters and weighting networks are selected by a 50 position switch either manually or by remote control, from another instrument. Use of impedance transformation enables both input and output to be connected to a wide range of terminating impedances.

The filter sets can also be operated from a 12 V DC supply, (e.g. car battery) for measurements in the field.

The 1/3 octave filter characteristics provide a considerable improvement over previous designs (peak-valley ripple is 0.5 dB). The filters conform to the recommendation laid down by the International Electrotechnical Commission (IEC 225 1966), the American Standard Association (ASA S1,11-1966 class III) and the German DIN 45652.

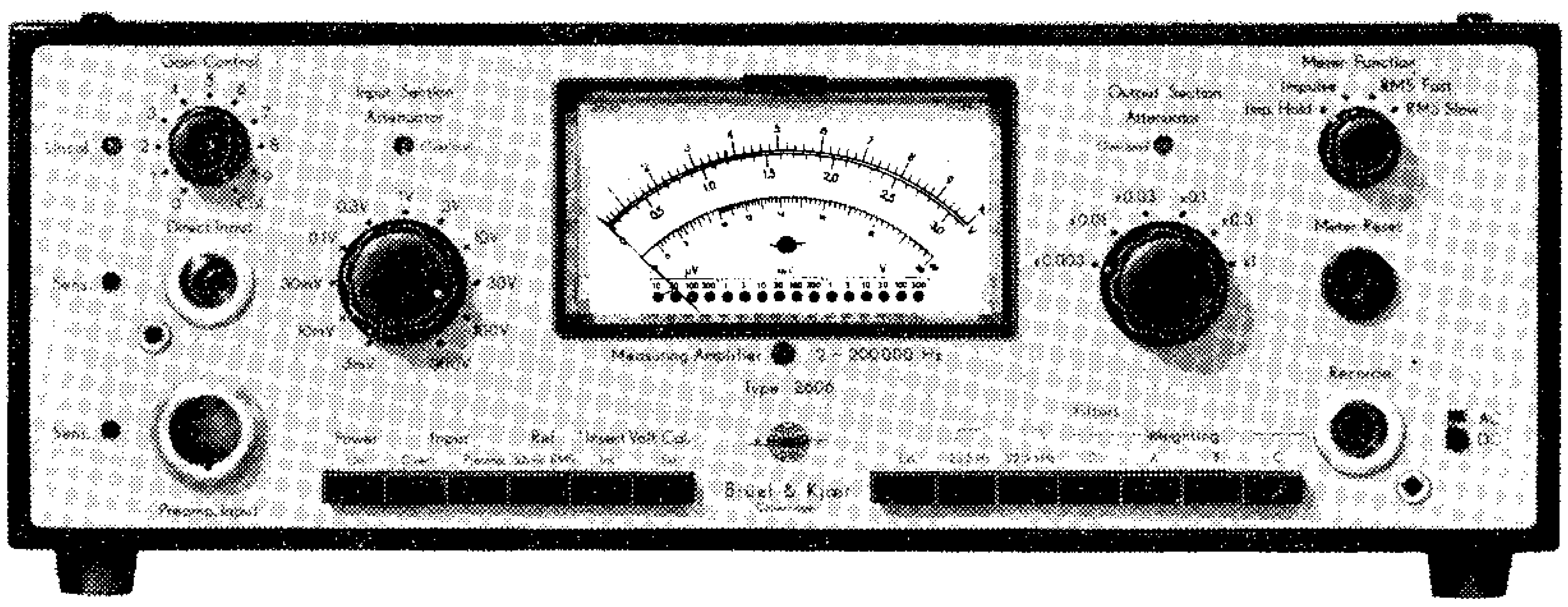
The octave filters are formed from three adjacent 1/3 octave filters. The switching is such as to provide octave filters with centre frequencies spaced one octave apart.



Measuring Amplifier Type 2606

The 2606 is a compact portable instrument capable of the complete range of sound and vibration measurements. It combines extreme versatility with a wide measuring range and laboratory precision.

It is basically a wide range voltmeter (2 Hz – 200 kHz), with interchangeable meter scales calibrated directly for sound and vibration measurements, and with selectable weighting networks and meter time constants.



Used with one of the B & q condenser microphones, the 2606 becomes a Precision Sound Level Meter to IEC Recommendation 179 and an Impulse Sound Level Meter to the proposed IEC recommendation, and conforming with the German DIN 45633 parts 1 and 2. The meter indicates true RMS for a wide range of signals and the meter circuit also features a maximum RMS hold capability.

Frequency analysis can be accomplished in conjunction with for instance the Filter Sets 1612, 1614/15, or the Heterodyne Slave Filter 2020.
Used in conjunction with the B & K accelerometers and preamplifiers, vibration level can be measured directly in the range 2 Hz – 40 kHz and from less than 0.001 g to several thousand g.
The 2606 in conjunction with the B & K 2617 preamplifier can be used for Insert Voltage calibration of microphones.
The 2606 can be powered from mains voltages from 100 V to 240 V AC, or from a 12 V DC supply. It can thus be used in the field for measurements of laboratory accuracy, powered from batteries.
A built-in oscillator supplies an accurate 5.6 V RMS, 1 kHz reference signal for calibration purposes.
An overall amplification of 114 dB is available between the input and recorder socket, enabling the 2606 to be used as a low distortion amplifier. Two attenuators provide attenuation over a 150 dB range in accurate 10 dB steps. This gives the instrument a voltage measuring range from 10 μV to 300 V full-scale. The attenuaor setting is displayed by indicator lamps on the meter scale.

The signal can be recorded either before (AC) or after (DC) rectification.
When the input or output amplifiers are overloaded, separate warning lamps light up.
The contacts of the relay which operates when the overload indicators are lit can be used, for example, for lifting the pen of a level recorder during overload.

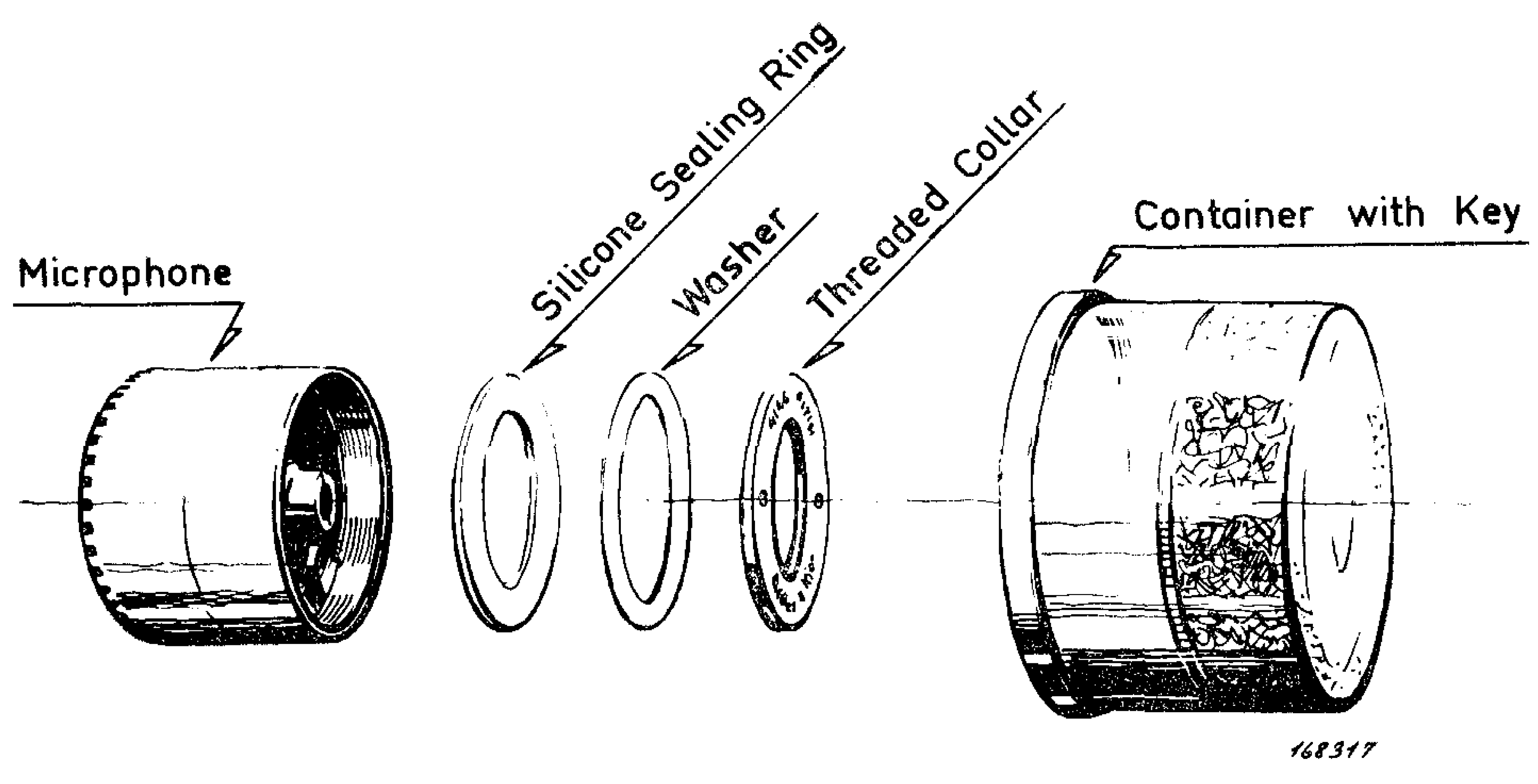
Condenser Microphone Type 4146

This microphone is a special version of the B & K 1" condenser microphones, adapted for low frequency measurements.

The microphone is identical in construction to the pressure microphone Type 4144. The low frequency range has been extended downwards by sealing the pressure equalization vent with a silicone-rubber ring tightened with a collar as shown in the figure. Thus the lower limiting frequency is adjusted to less than 0.1 Hz.

Used with the B & K Microphone Carrier System Type 2631, the microphone makes up a versatile system for measurements over the complete frequency range of the microphone. The Carrier System can measure down to DC.

For sound measurements in the range 2 Hz upwards the microphone can be used just as any other 1" condenser microphone in connection with any B & K measuring and analyzing instruments. The rubber sealing ring may be removed to give a normal cut-off in the range 1–2 Hz.



Microphone Carrier System Type 2631

This system has been designed for sound measurement in the frequency range 0 to 150,000 Hz. It is especially useful for measuring low frequency pressure variation, such as those found in sonic booms, thunderstorms, wind tunnels and pressure chambers.

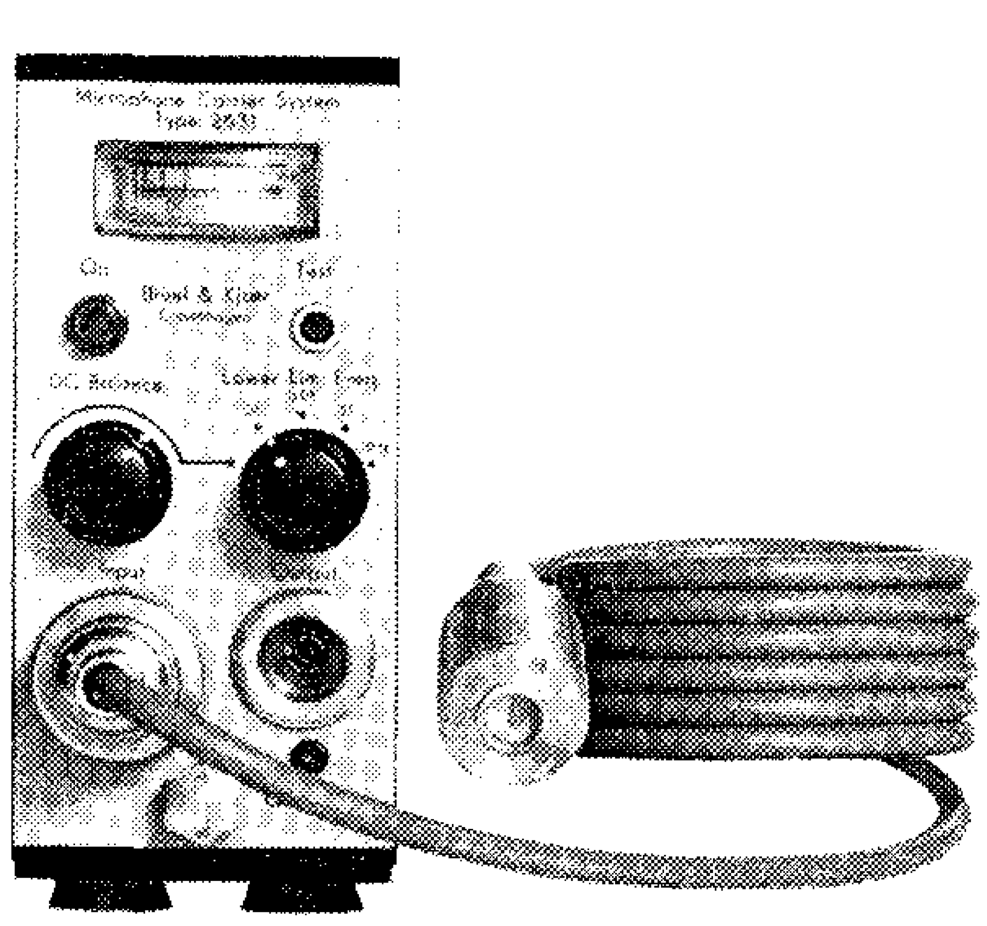
In sonic boom measurements, for example, a wide bandwidth is required if an accurate representation of the pulse is to be recorded.

The microphone carrier system opens up new possibilities for very low frequency sound measurements which were not possible before, due to the limitations of conventional preamplifiers.

Special condenser microphones are needed to realize the very low frequency response for sound measurement, because normal microphones have a pressure equalization vent which limits the low frequency response, usually to some 1 to 3 Hz. The B & K type 4146 condenser microphone i sealed, and gives a lower limit of less than 0.1 Hz. Microphones 4144 and 4145 can also be sealed to give a similar characteristic.

The output of the microphone carrier system can be connected to any recording or display instrument with an appropriate frequency range and an input impedance greater than 1.2 kΩ.
In connection with the B & K Tape Recorder 7001 a very stable and sensitive system is obtained, suitable for storing information for later analysis. Using the frequency transformation technique with the Tape Recorder and a 1/3 octave filter set, analysis of sound is possible down to 0.05 Hz with single transformation.

Cables of considerable length may be used between the microphone and power supply and between the power supply and the recording instrument, subject to the limitation of the 10 mA output current.

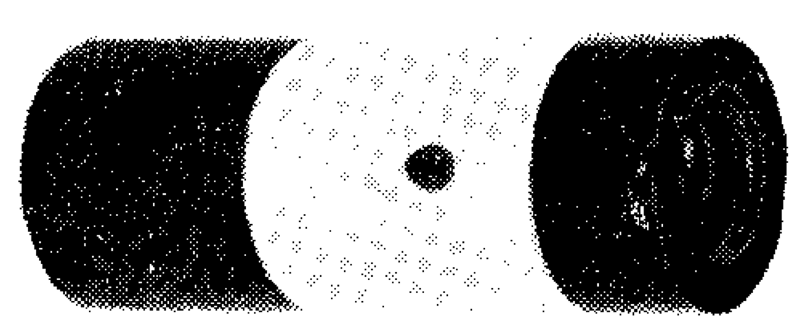


2631 1:4

Sound Level Calibrator Type 4230
This calibrator enables easy calibration of sound measuring instrumentation in the field.

The calibration frequency is 1000 Hz, (the reference frequency for the standardized international weighting networks), so the same calibration value is obtained for all weighting networks (A, B, C, D and Linear). The calibration pressure of 94 $\pm$ 0.25 dB re 2×10^{-5} N/m^2 is equal to 10 μbar or 1 N/m^2, convenient for calibration.

The principle of operation of the calibrator is: A stabilized 1000 Hz oscillator feeds a piezoelectric driver element which vibrates the diaphragm and creates the sound pressure level in the coupler volume. The system is driven at its resonant frequency, at which the equivalent coupler volume is more than 200 cm^3. The sound pressure is therefore independent of the microphone's volume and the accuracy with which it is connected to the calibrator, an added assurance for non-experts.



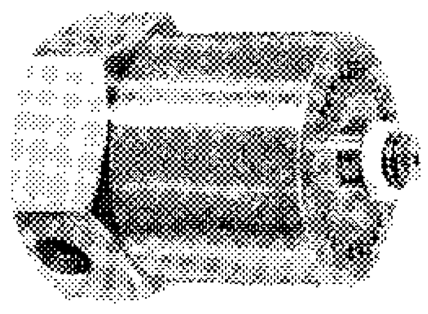
Level Recorder Type 2305
The level recorder has been redesigned using all solid state electronic circuits. This results in less power requirements and less heat dissipiation. Other specifications are purposely kept the same.

Piezoelectric Accelerometer Type 4338

The 4338 is suitable for a wide variety of applications where a high sensitivity accelerometer is required. It features a new "inverted single-ended" compression" design which produces the desirable accelerometer characteristics of low sensitivity to strain, temperature variations, acoustic noise, magnetic fields and transverse vibration, whilst having a sensitivity as high as 100 pC/g in the principal axis.

The robust, sealed construction allows operation in extreme environmental conditions, and provision is made for water cooling for operation on hot surfaces up to 1000°C.

An individual calibration chart accompanies each unit, the stability of calibration being assured by a temperature stabilizing procedure which makes the 4338 suitable as a laboratory standard.



Deviation Bridge Type 1519

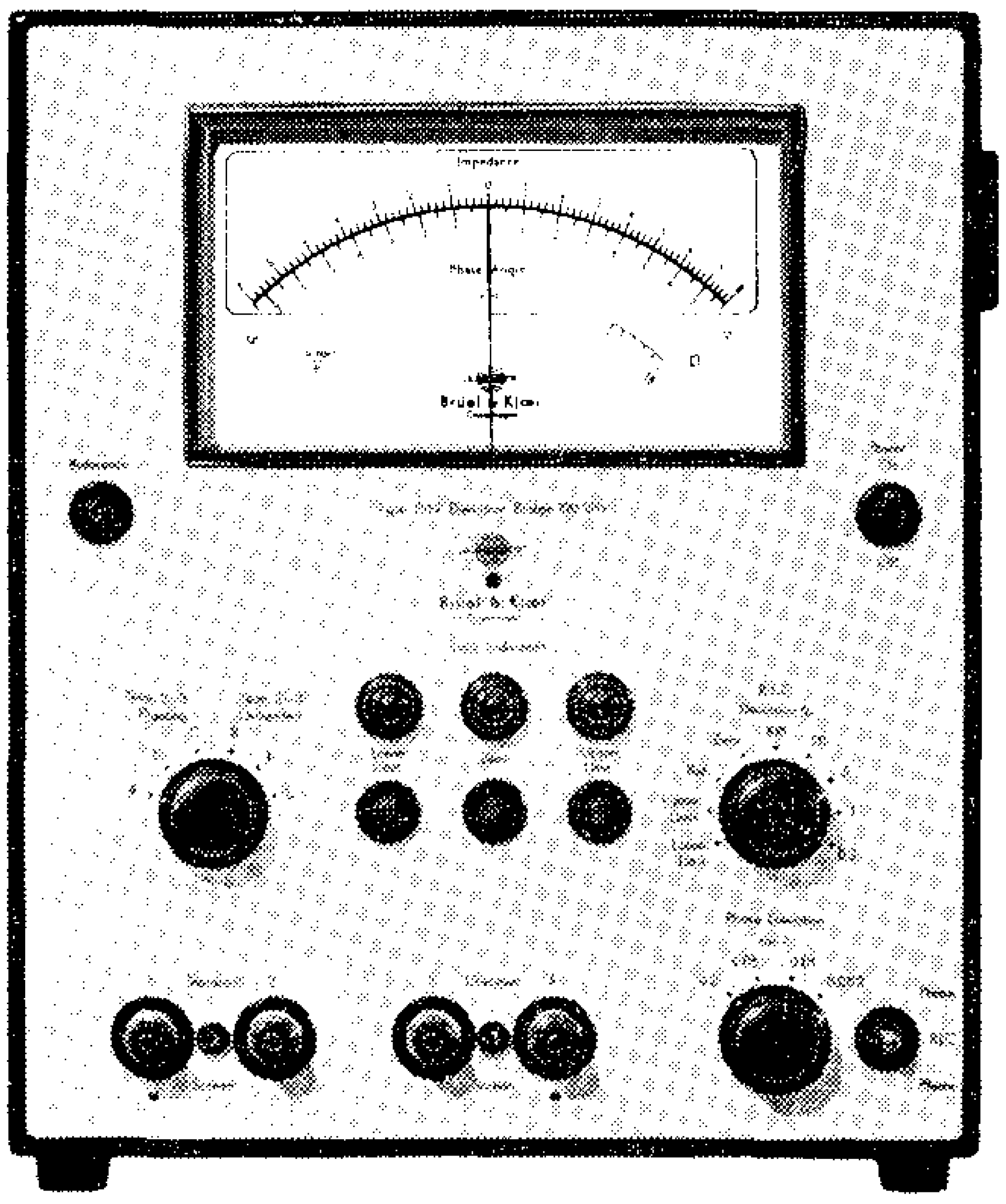
Type 1519 is a solid state sucessor to the Type 1506 deviation bridge.

The design shows a number of improvements a few, of which, can be mentioned:

Measuring ranges are extended to 0.3 % impedance deviation and to tan δ = 0.003 for full scale deflection.

Impedance ranges and phase angle ranges are selected by separate fixed attenuators. Thus the calibration of each measuring range is kept fixed.

Built in tolerance indicator lamps can be supplemented by external lamps. This feature in conjunction with the analog DC output gives great versatility in servo-directed sorting arrangements.



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